

AXIAL SOLID-DENSITY DISTRIBUTION IN
GAS SOLID FLUIDIZED BEDS

by

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INTRODUCTION

The fluidization technique has been employed widely in many fields of the process industry. The term fluidization generally implies that solid particles are suspended in a fluid flowing through a vessel. The mixture of particles and fluid, therefore, forms the over-all bed.

The advantages and disadvantages of the fluidization technique in comparison with the conventional packed bed reactors can be found elsewhere. (11, 28, 25). May (15) in his recent paper pointed out, "Chemical reactions with fluidized beds have shown efficiencies varying all the way from 'better than fixed beds' to as low as 5% of fixed beds. The low figure shows that the bypassing must be important."

Wilhelm and Kwauk (26) first observed and described the distinction between particulate fluidized bed and aggregative fluidized bed. In the particulate fluidization, particles are discretely separated from each other and a mean free path for the particles seems to exist; whereas, the particles in the aggregative fluidization are present in the bed not as individual units, but as local aggregates. The particulate fluidized bed resembles closely an ideal expansion of a fixed bed, whereas an aggregative fluidized bed is a heterogeneous body composed of two separate coexisting phases --- dense phase and bubble phase (dilute phase). A close analogy can be found between the characteristics of a fixed bed and a particulate fluidized bed, but it is not so between a fixed bed and an aggregative fluidized bed. The differences are chiefly due to the fact that:

- a) Particle distribution in the aggregative bed is extremely heterogeneous.
- b) There is no definite way to measure the height of bed expansion.

Literature Survey*

During the past years, the characteristics of aggregative fluidized beds have been studied by several different experimental methods. The experimental methods used by previous workers are summarized as follows:

1. High speed photographic studies were used by Matheson et.al.(14), Furukawa and Omae (7), and by Leopoldo Massimilla and J. W. Westwater (13). They took high speed photographs of bubbles through transparent walls of fluidized columns for gas-solid systems.

2. Shuster and Kisliak (24) studied the uniformity of aggregative fluidized beds by using diaphragm plates to measure pressure gradients.

3. By measuring the variation of the electric capacitance between condenser probes, Morse and Ballou (18), and J. M. Dotson (5) examined the uniformity of aggregative fluidization. Later Bakker and Heertjes (1, 2) applied a similar technique to measure the point porosity of fluidized beds at various locations.

4. Yasui and C. N. Johanson (27) devised a small light probe to study the frequency, thickness and the rising speed of bubbles in aggregative fluidized beds.

5. The X-ray absorption technique made it possible for Grohse (9) to measure the over-all density of the aggregative fluidized bed without disturbing the bed. Baumgarten and Pigford (3) employed a γ -ray absorption technique in the study of bubble size and bubble frequency in aggregative fluidized beds.

* A rather complete literature survey concerning fluidization can be found in the Ph.D. thesis by P. T. Shannon (22).

Some of these investigators have proposed certain mathematical or empirical models to fit their experimental data. None of these models has as yet been verified as acceptable and applicable to aggregative fluidized beds in general.

Purpose

The major purpose of the present work was to investigate the axial distribution of the fluidized bulk density in an aggregative fluidized bed by γ -ray technique. This is to determine qualitatively and quantitatively the axial density profile as the function of several operational variables.

EXPERIMENTAL

Application of γ -ray Absorption Technique to a Fluidized Bed

The radiation attenuation method of measuring density of a two-phase fluid has become prevalent during the past years (19, 3). This method of measuring density is based on the absorption of high energy radiation from a radioactive source which can be detected by a scintillation counter probe. Two of the characteristics of γ -rays are their high energy and their short wave length. When it is applied to the measurement of density, it is essentially a process of counting the number and indicating the kind of atoms by which the radiation is being absorbed. The attenuation of γ -rays is virtually independent of the chemical or physical state of the atoms.

If the absorption of the primary radiation by air (or by other fluidizing gas) is negligible, the attenuation of γ -rays shown on the scintillation probe is due to the absorption of γ -rays by the thickness of the walls of the fluidizing column, the constant absorbing media, such as the protective enclosure of the scintillation crystals (Fig. 1), and the particles packed or fluidizing in the path between the radioactive source and the scintillation counter probe. The γ -rays describe an essentially parallel path in passing from the source to the scintillation counter probe. The γ -rays absorption by a homogeneous material of density, ρ , and thickness, D , can be described by the Lamberts-Beer's Law:

$$I = I_0 e^{-\mu_m \rho D}$$

or

$$\ln (I/I_0) = -\mu_m \rho D \quad (1)$$

The absorption coefficient " μ_m " varies in accordance with the following relationship:

$$\mu_m = (N_a/M) \sigma$$

where,

N_a is avagadro's number

M is the atomic weight, and

σ is the microscopic absorption cross section.

Therefore, the line average of solid density in the fluidized bed can be measured for each axial position by recording the radiation attenuation at each position and by converting it to density by equation (1), provided the following assumption is made:

The fluidizing particles and the fluidizing gas together form a substance of density, ρ , and with a thickness, D_c (the inside diameter of the fluidizing column), where the absorption of γ -rays by the air (or fluidizing gas) is negligible.

Apparatus and Equipment

A photograph of the equipment is shown in Fig. 1 while a schematic diagram is presented in Fig. 2. The equipment may be divided into two categories; (1), the column and its accessories, and (2), the gamma-ray source and nuclear instrumentation for detection, measurement, and recording of the transmitted gamma-ray beam.

The Column and Accessories. A 24-inch-high, 3.97-inch-diameter, lucite column was used. The column was covered at the top with a fine screen to permit the exhaust of the fluidizing air, but to retain any particles that might have been fluidized as high as the top of the column. The lower flange of the column was equipped with a pressure tap for measuring the pressure drop over the bed. Other taps were located every four inches up the flange of the column. The column was bolted directly to the distributor assembly.

Accessories to the column included the distributor assembly for distributing the air, manometers for measuring the pressure drops over the column and across the distributor, an air supply to provide dry air at a known velocity, and a jack for raising and lowering the column so that different heights in the bed could be investigated. These accessories will be discussed in this order.

Distributor Assembly. The air was distributed by the distributor assembly shown in Fig. 3. The air first passed through an 8-inch calming section of quarter-inch ceramic spheres. Above the bed of ceramic spheres, the air was distributed by a canvas filter cloth positioned between the lower flange of the column and a second lucite flange similar to the lower flange of the column.

Manometers. A 20-inch manometer using a manometer oil with specific gravity of 0.818 was used for measuring the pressure drop over the column.

Air Supply. A compressed-air line from the Kansas State University Physical Plant provided air for fluidizing. The air was dried by passing it through a silica-gel air dryer. The flow rate of air was measured by a rotameter.

Jack Assembly. The column-distributor assembly was mounted on an iron frame which was raised and lowered by means of a hand jack making it possible to investigate various heights in the column.

Gamma-ray Source and Nuclear Instrumentation. Two gamma-ray sources were used to provide a greater intensity, thereby increasing the count rate and improving the statistics. These two sources were a five millicurie Ra^{226} source and a twenty-five millicurie Cs^{137} source. The two sources were placed in a 5/8-inch hole drilled lengthwise in a 2 x 4 x 8-inch lead brick. The 5/8-inch hole served as a collimator for the γ -radiation from the two sources. For added shielding, other lead bricks were placed around the brick containing the source. The beam was directed through the center of the column. On the opposite side of the column a 7/16-inch diameter collimator transmitted the beam to the scintillation probe which was positioned directly behind the collimator. This collimator consisted of two bricks. This was done

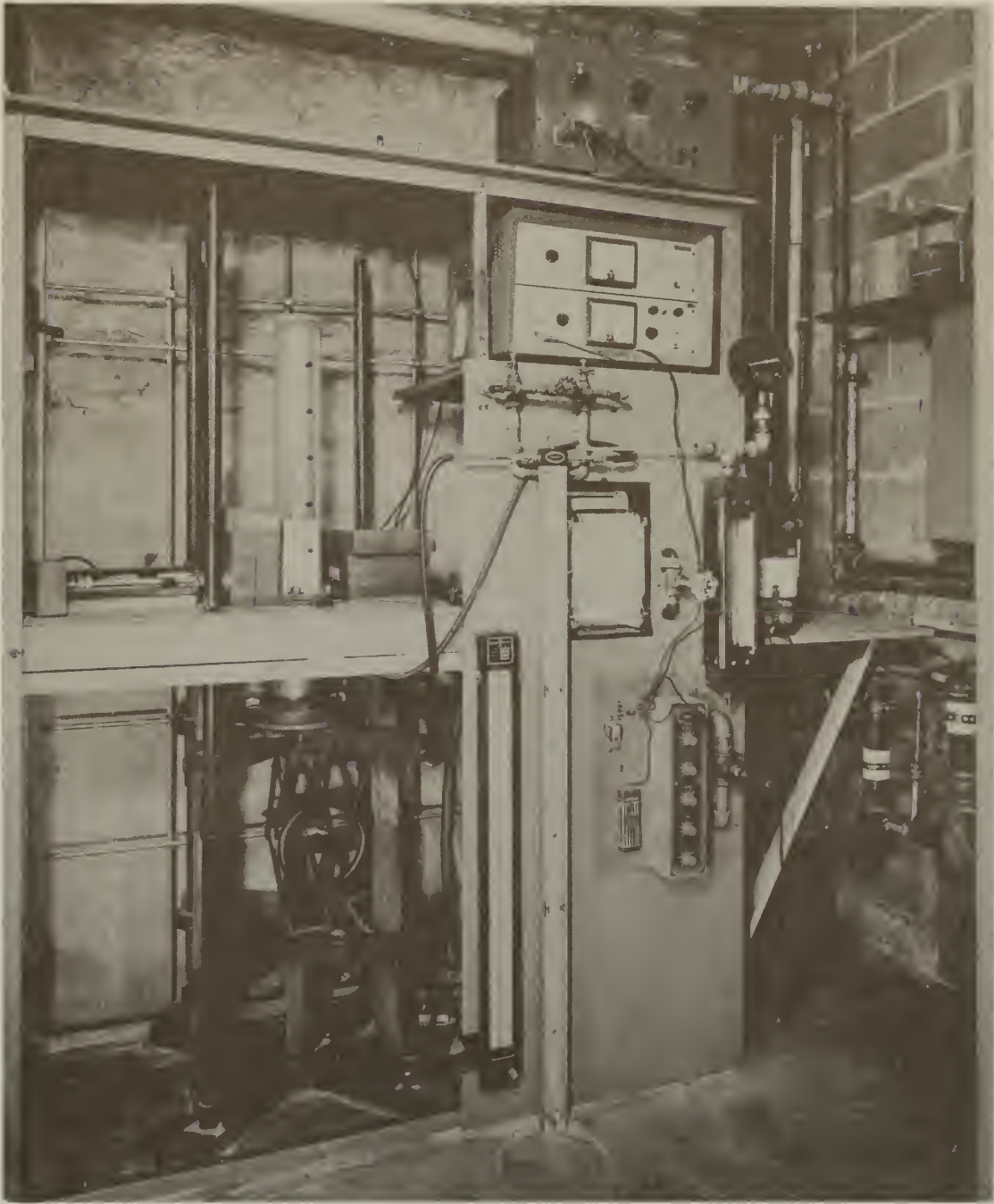


Figure 1. Photograph of column and associated equipment.

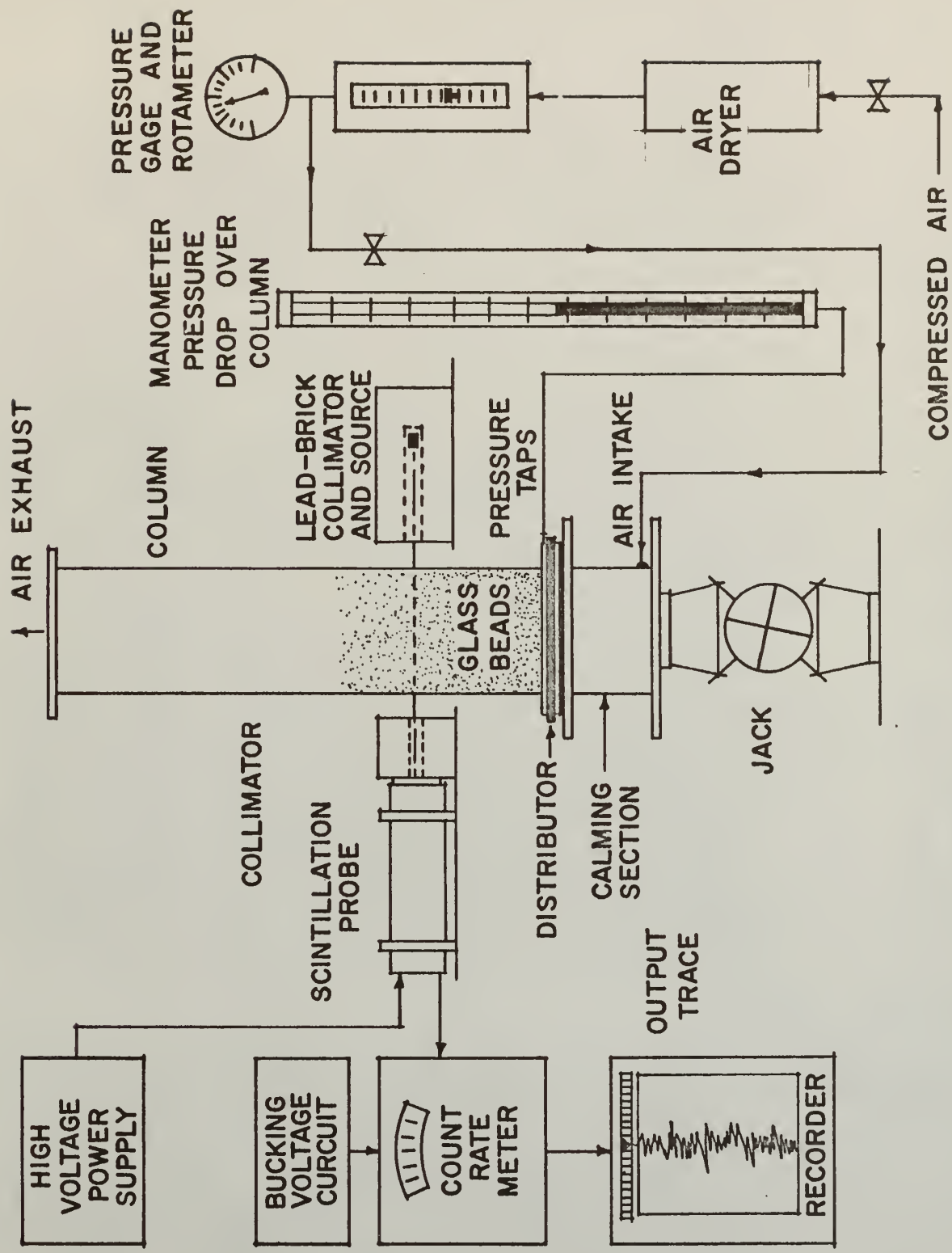


Fig.2. Schematic diagram of column, accessories, and instrumentation.

so that the 8-inch side of the bricks could be used to shield the probe from stray radiation. A close-up view of the source, collimators, column, and probe is shown in Fig. 4.

Instrumentation for the measurement, detection, and the recording of the transmitted gamma-radiation included a scintillation probe, power supply, modified count rate meter, bucking voltage supply, and recorder. These instruments will be discussed in this order.

Scintillation Probe. The probe, a Model Dp3, B-J Electronics, Borg-Warner Corporation, was used without modification. This instrument consisted of a Type 704, thallium-activated sodium iodide scintillation crystal, a Type 6292, DuMont photomultiplier tube, and a one-tube (6AK5) preamplifier. The probe was operated at 1200 volts.

Power Supply. A John Fluke Mfg. Co. Model 400BDA high-voltage power supply was used for operation of the scintillation probe.

Modified Count Rate Meter (CRM). The scintillation probe output was fed into a B-J Electronics Model DM1-D count rate meter that had been slightly modified. The meter included five time constants, 0.3, 1, 3, 10, and 30 seconds. Only the 30-second time constant was used for measurement of the average line density. The rate meter was equipped with five scale multiplication constants; 10, 30, 100, 300, and 1000. 300 scale was chosen since it gave the highest sensitivity. In choosing the 300 scale multiplier it became necessary to modify the rate meter. With the high activity used, it was necessary to apply a bucking voltage in order that the CRM would read on-scale, and so that the full range of density fluctuations could be covered. The bucking voltage was added to the input of the vacuum-tube voltmeter circuit of the CRM. This bucking voltage acted as a negative bias on the input grid of the voltmeter circuit.

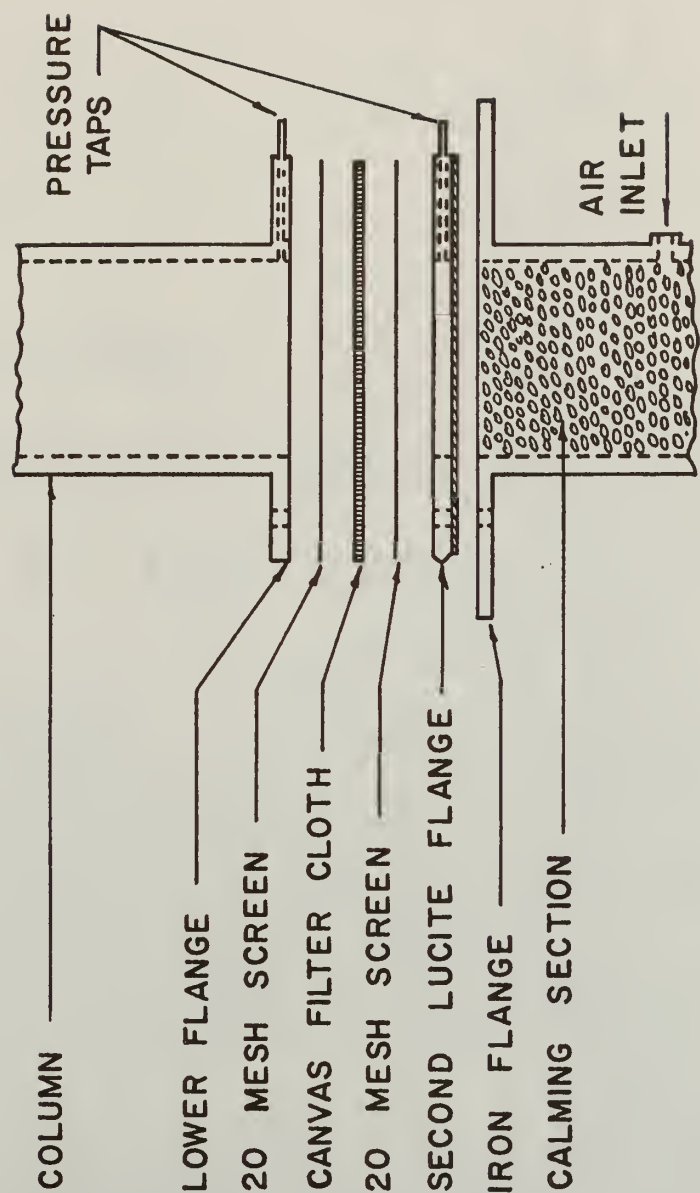


Fig. 3 Distributor assembly.

Recorder. This instrument, an Esterline-Angus Model AW graphic ammeter was used without modification. The recorder was connected in series with the CRM.

Preparation of Fluidizing Particles

The particles used in this experiment were glass beads manufactured by Minnesota Mining and Manufacturing Co. In order to minimize the error due to the size distribution effect when doing the investigation for narrow-cut particles, the purchased glass beads were reclassified by a sieving machine.

The glass beads actually used in the experiment were of two sizes:

(1) 40-45 U. S. mesh: particle diameter ranges from 0.0138 inches to 0.0164 inches, the average diameter of this size is 0.0153 inches.

(2) 80-100 U. S. mesh: particle diameter ranges from 0.0049 to 0.0070 inches, the average diameter of this size is 0.00642 inches.

The particles of 40-45 mesh were obtained by shaking about 100 grams of 3M catalog No. 70 glass beads on the sieving machine for three minutes, those particles passing through 40 mesh screen and retained on the 45 mesh screen were collected.

The 80-100 mesh particles were obtained from the catalog No. 110 glass beads by using the same technique as for 40-45 mesh particles. Those passing through 80 mesh and retained on 100 mesh particles were collected.

Some additional data were obtained with 60-70 mesh particles in order to confirm some of the observations and correlations resulting from the use of the other two particle sizes.

Preliminary Calibrations

Count Rate Meter vs. Solid Density in the Fluidized Bed. For the preliminary calibration, spacers made from very thin aluminum sheets were used, which had inside air-gap widths of 0.25, 0.50, 0.75, 1.00, 1.25, 1.50 & 2.00 inches, respectively.

The calibration was done by inserting perpendicularly one spacer at a time into the densely packed bed of glass beads of density equal to 1.502 grams/cu-cm, and then recording the radiation attenuation. This process was also repeated by packing the glass beads inside the spacers, and leaving the area surrounding the spacers empty. Then the density corresponding to a particular spacer could be calculated with the following two relations:

- (a) (Density of packed bed) $\times (D_c - D_s)/D_c$, when the glass beads were outside the spacers.
- (b) (Density of packed bed) $\times (D_s/D_c)$, when the glass beads were inside the spacers.

These two linear relationships can be proved by the combined Lambert's equation:

$$\ln(I/I_0) = (\mu_m)_c \rho_c d_c + (\mu_m)_{Al} \rho_{Al} d_{Al} + (\mu_m)_p \rho_{pa} d_p \quad (2)$$

Where the subscript "c" denotes the radiation attenuation due to γ -rays absorption by the walls of column; subscript "Al", by the aluminum sheets and subscript "p", by the packed particles inside spacers or in the annular space between column walls and the spacer sheets. Care was taken that the attenuation due to column wall, aluminum sheets and all other possible absorbing media stayed constant through the calibration (Fig. 4 and 5). Therefore, Equation (2) can be simplified as:

$$\ln(I/I_0) = \text{constant} + (\mu_m)_p \rho_{pa} D_p^i \quad (3)$$

where $(\mu_m)_p \rho_{pa}$ is also constant throughout the calibration, since the same glass beads were used, and the density of solid glass remained constant. Then the change in $\ln(I/I_0)$ is a linear function of D_p^i .

In dealing with the density of a fluidizing state, the radiation attenuation reading corresponding to one of the spacer calibrated values should have a density corresponding to the mass of $\rho_{pa} D_p^i A_r$ particles distributed homogeneously in the volume, $D_c A_r$, where,

A_r = projected area of γ -beam path.

Therefore, the density of the fluidized bed at a certain attenuation reading is:

$$\rho_f = \frac{\rho_{pa} D_p^i A_r}{D_c A_r} = \frac{\rho_{pa} D_p^i}{D_c} \quad (4)$$

where, $D_p^i = D_c - D_s$, when the beads were outside the spacer.

$D_p^i = D_s$, when the beads were inside the spacer.

As described in the section of Apparatus and Equipment, the radiation attenuations were measured and recorded on the graphic ammeter. In using the 300 scale multiplier of the count rate meter, it was necessary to calibrate the density vs. recorder reading in the manner mentioned above for several different bucking voltages. This was done so that the full range of densities, i.e., from empty column to packed bed, could be covered.

A complete correlation chart of recorder reading vs. density of bed is given in Table I. In the table:

300 I scale is for B. V. of 8170--8300 v

300 Ia scale is for B. V. of 9870--10010 v

300 II scale is for B. V. of 11300--11450 v

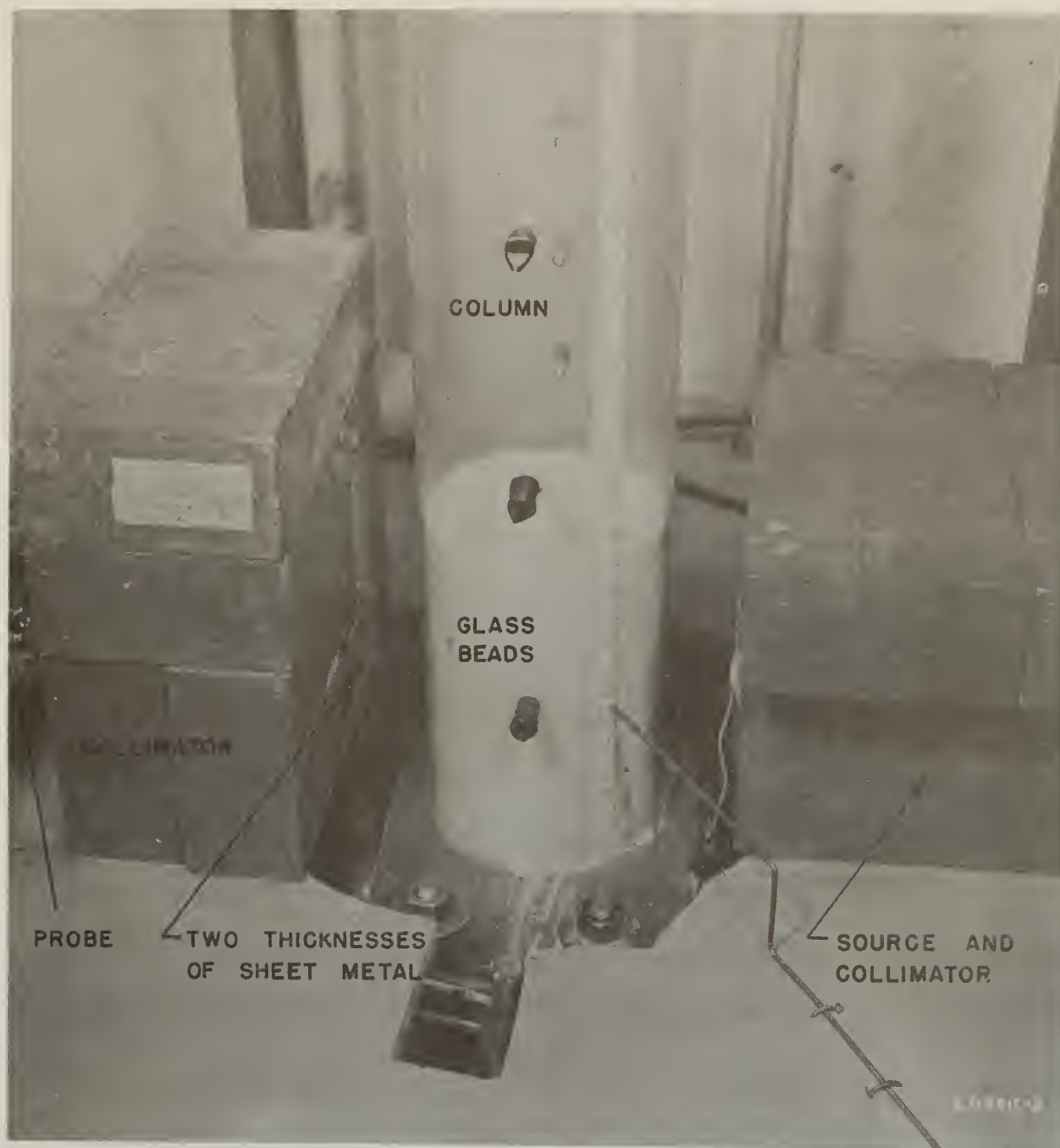


Figure 4. Bed arrangement with spacer out.

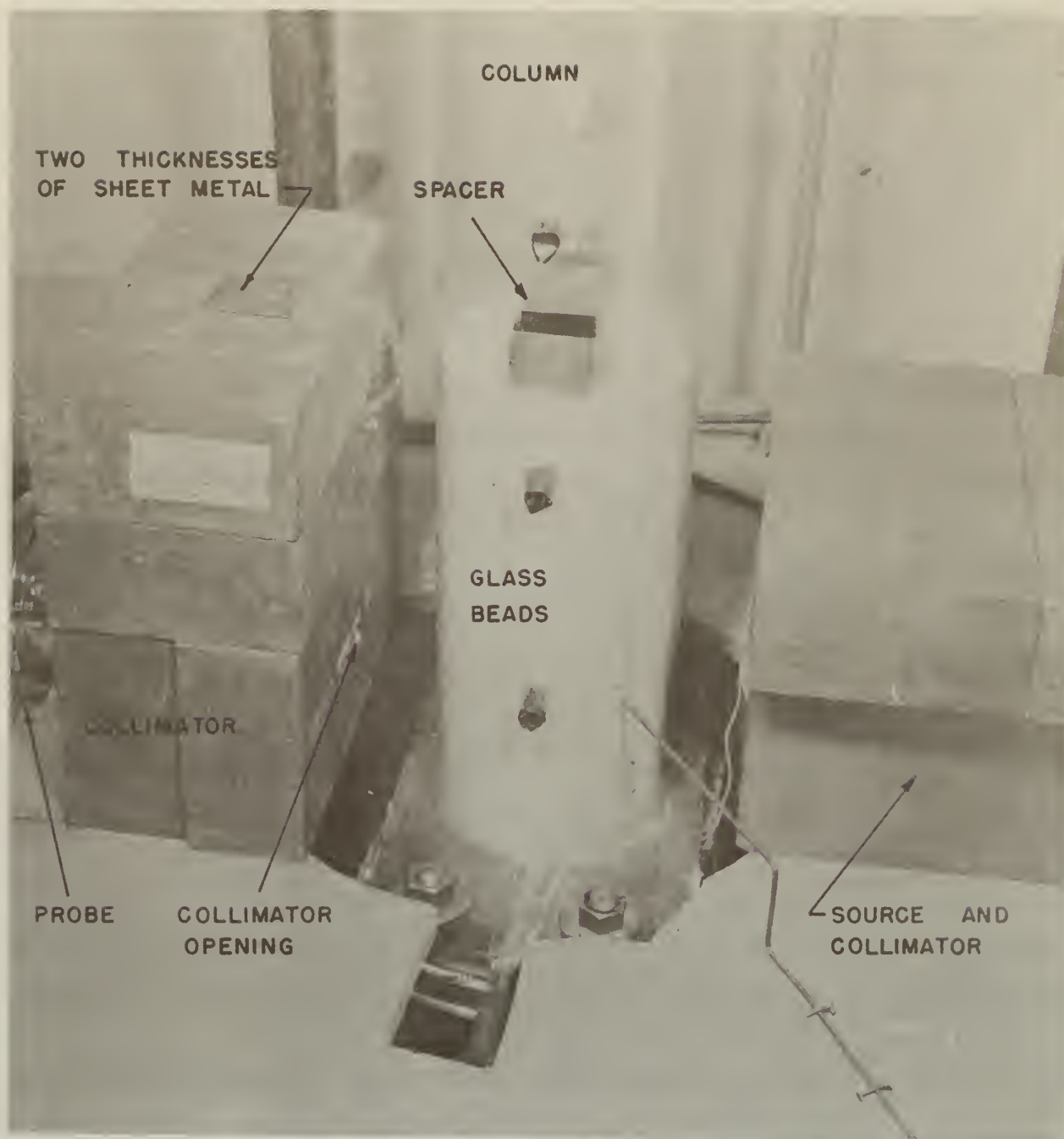


Figure 5. Calibration technique with spacer inserted.

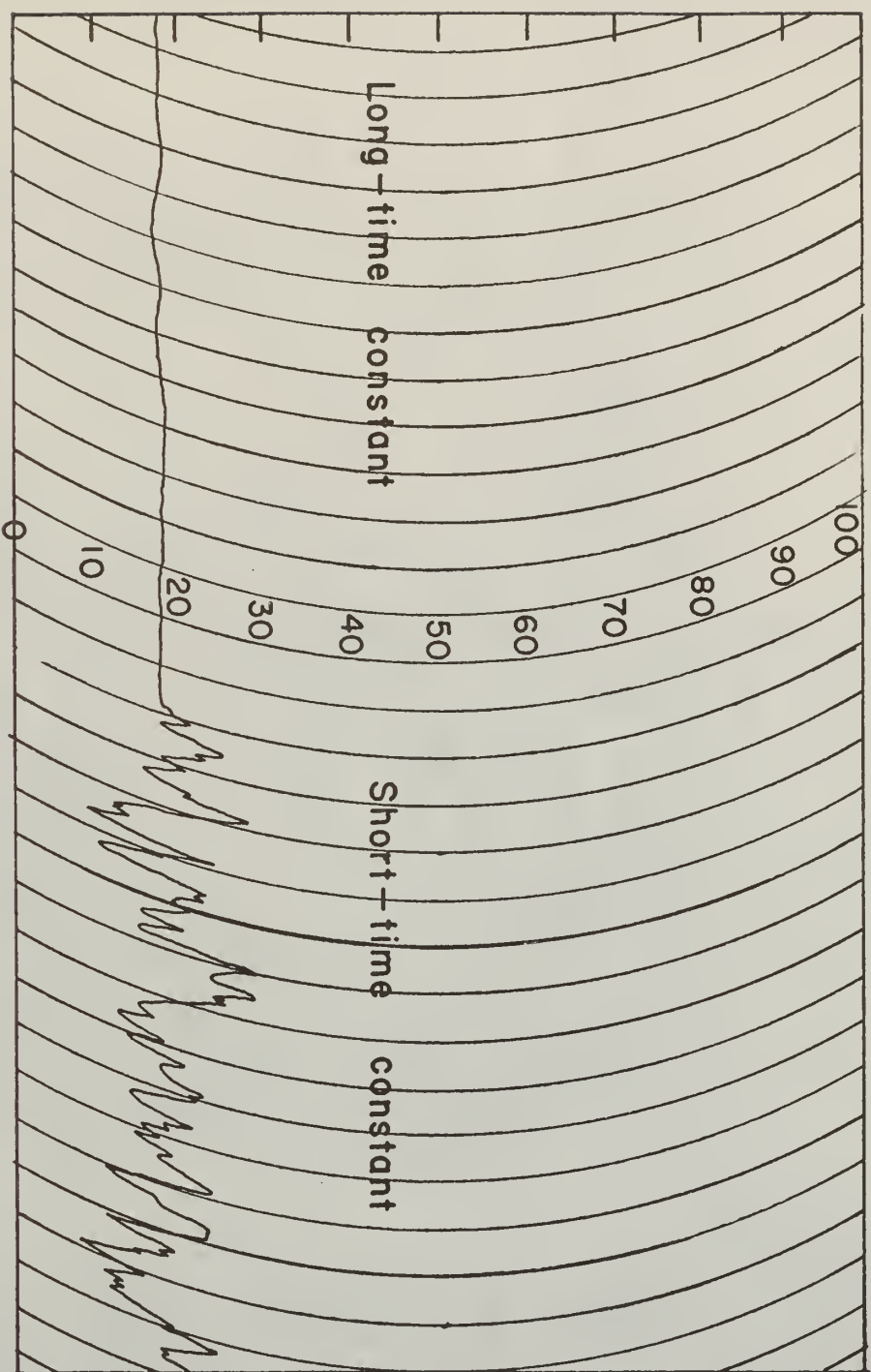


Fig. 6. Typical recorder trace for packed bed on 300 I scale.

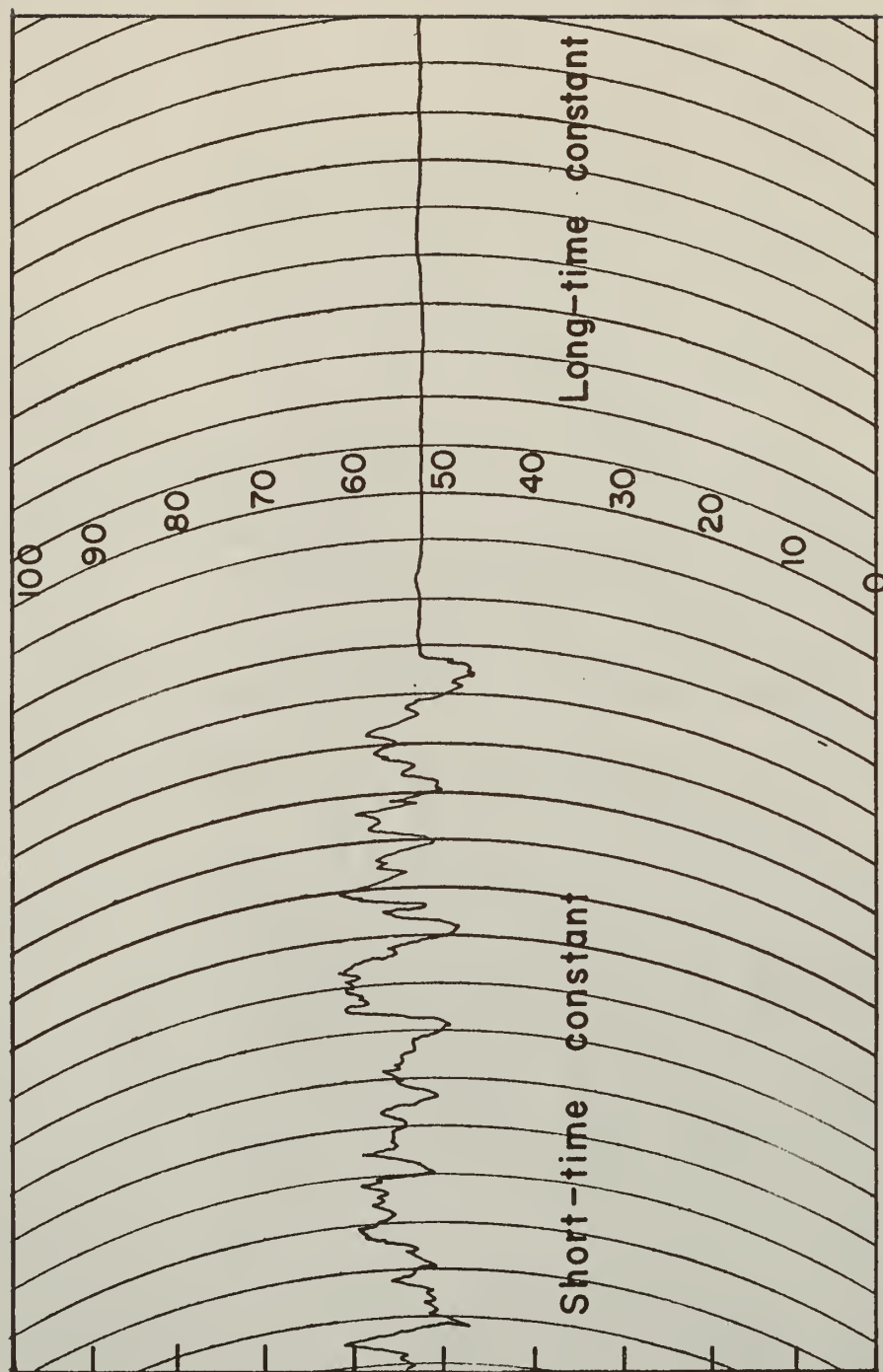


Fig. 7. Typical recorder trace for empty column on 300 III scale.

Table I. Count Rate Meter (CRM) Calibration.

300 I Scale

CRM Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm
001	1.6300	026	1.4550	051	1.2600	076	1.0260
002	1.6200	027	1.4460	052	1.2500	077	1.0150
003		028	1.4400	053	1.2420	078	0.0050
004	1.6100	029	1.4330	054	1.2330	079	0.9940
005		030	1.4250	055	1.2250	080	0.9830
006	1.5950	031	1.4170	056	1.2160	081	0.9720
007		032	1.4100	057	1.2080	082	0.9610
008	1.5800	033	1.4020	058	1.2000	083	0.9490
009		034	1.3950	059	1.1900	084	0.9370
010	1.5700	035	1.3880	060	1.1810	085	0.9260
011		036	1.3800	061	1.1720	086	0.9150
012	1.5500	037	1.3720	062	1.1630	087	0.9040
013		038	1.3650	063	1.1550	088	0.8910
014	1.5350	039	1.3580	064	1.1450	089	0.8780
015		040	1.3500	065	1.1350	090	0.8660
016	1.5250	041	1.3420	066	1.1260	091	0.8540
017		042	1.3340	067	1.1160	092	0.8410
018	1.5100	043	1.3260	068	1.1070	093	0.8260
019		044	1.3180	069	1.0980	094	0.8110
020	1.5000	045	1.3100	070	1.0880	095	0.7970
021		046	1.3020	071	1.0780	096	0.7820
022	1.4850	047	1.2940	072	1.0680	097	0.7660
023		048	1.2850	073	1.0580	098	0.7510
024	1.4700	049	1.2760	074	1.0480	099	0.7350
025		050	1.2680	075	1.0370	100	0.7170

Table I. Count Rate Meter (CRM) Calibration. (Cont.)

300 Ia Scale

CRM Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm
001	1.1270	026	0.9500	051	0.7510	076	0.5030
002	1.1200	027	0.9430	052	0.7430	077	0.4920
003	1.1140	028	0.9360	053	0.7340	078	0.4810
004	1.1080	029	0.9280	054	0.7250	079	0.4710
005	1.1000	030	0.9190	055	0.7150	080	0.4600
006	1.0940	031	0.9110	056	0.7050	081	0.4500
007	1.0870	032	0.9040	057	0.6950	082	0.4390
008	1.0820	033	0.8960	058	0.6850	083	0.4280
009	1.0740	034	0.8900	059	0.6750	084	0.4170
010	1.0670	035	0.8840	060	0.6650	085	0.4060
011	1.0600	036	0.8760	061	0.6550	086	0.3950
012	1.0510	037	0.8700	062	0.6450	087	0.3840
013	1.0430	038	0.8620	063	0.6360	088	0.3730
014	1.0350	039	0.8540	064	0.6260	089	0.3620
015	1.0280	040	0.8450	065	0.6160	090	0.3500
016	1.0210	041	0.8370	066	0.6060	091	0.3400
017	1.0140	042	0.8300	067	0.5960	092	0.3300
018	1.0070	043	0.8210	068	0.5860	093	0.3190
019	1.000	044	0.8130	069	0.5760	094	0.3070
020	0.9920	045	0.8050	070	0.5660	095	0.2950
021	0.9860	046	0.7960	071	0.5560	096	0.2830
022	0.9800	047	0.7870	072	0.5450	097	0.2700
023	0.9720	048	0.7780	073	0.5350	098	0.2560
024	0.9650	049	0.7700	074	0.5240	099	0.2400
025	0.0570	050	0.7610	075	0.5140	100	0.2250

Table I. Count Rate Meter (CRM) Calibration. (Cont.)

300 II Scale

CRM Reading :	Density : gm/cu-cm :	CRM : Reading :	Density : gm/cu-cm :	CRM : Reading :	Density : gm/cu-cm :
001	0.8860	026	0.6940	051	0.4810
002	0.8800	027	0.6860	052	0.4720
003	0.8730	028	0.6780	053	0.4630
004	0.8660	029	0.6700	054	0.4540
005	0.8590	030	0.6620	055	0.4450
006	0.8510	031	0.6540	056	0.4350
007	0.8430	032	0.6450	057	0.4250
008	0.8360	033	0.6360	058	0.4150
009	0.8280	034	0.6260	059	0.4050
010	0.8210	035	0.6180	060	0.3950
011	0.8140	036	0.6100	061	0.3850
012	0.8060	037	0.6020	062	0.3650
013	0.7980	038	0.5950	063	0.3550
014	0.7900	039	0.5860	064	0.3460
015	0.7820	040	0.5770	065	0.3360
016	0.7750	041	0.5690	066	0.3260
017	0.7660	042	0.5610	067	0.3150
018	0.7580	043	0.5530	068	0.3050
019	0.7500	044	0.5440	069	0.2950
020	0.7420	045	0.5350	070	0.2840
021	0.7350	046	0.5250	071	0.2730
022	0.7260	047	0.5160	072	0.2620
023	0.7180	048	0.5070	073	0.2510
024	0.7100	049	0.5000	074	0.2400
025	0.7010	050	0.4900	075	0.2300
				076	0.2300
				077	0.2190
				078	0.2090
				079	0.1970
				080	0.1850
				081	0.1750
				082	0.1640
				083	0.1530
				084	0.1420
				085	0.1310
				086	0.1200
				087	0.1090
				088	0.9070
				089	0.0850
				090	0.0740
				091	0.0640
				092	0.0530
				093	0.0400
				094	0.0290
				095	0.0160
				096	0.0050
				097	0.0
				098	0.0
				099	0.0
				100	0.0

Table I. Count Rate Meter (CRM) Calibration. (Cont.)

300 IIA Scale

CRM Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm	CRM : Reading	Density : gm/cu-cm
001	0.5940	026	0.4400	051	0.2640	076	0.0550
002	0.5880	027	0.4330	052	0.2560	077	0.0450
003	0.5820	028	0.4260	053	0.2480	078	0.0370
004	0.5760	029	0.4200	054	0.2400	079	0.0280
005	0.5700	030	0.4130	055	0.2330	080	0.0200
006	0.5650	031	0.3060	056	0.2250	081	0.0110
007	0.5590	032	0.3990	057	0.2170	082	0.0030
008	0.5540	033	0.3910	058	0.2100	083	0.0000
009	0.5480	034	0.3850	059	0.2010	084	0.0000
010	0.5420	035	0.3780	060	0.1920	085	0.0000
011	0.5350	036	0.3700	061	0.1840	086	0.0000
012	0.5300	037	0.3640	062	0.1750	087	0.0000
013	0.5240	038	0.3580	063	0.1660	088	0.0000
014	0.5170	039	0.3510	064	0.1580	089	0.0000
015	0.5110	040	0.3440	065	0.1500	090	0.0000
016	0.5050	041	0.3360	066	0.1420	091	0.0000
017	0.4990	042	0.3290	067	0.1340	092	0.0000
018	0.4920	043	0.3220	068	0.1250	093	0.0000
019	0.4860	044	0.3150	069	0.1150	094	0.0000
020	0.4800	045	0.3090	070	0.1060	095	0.0000
021	0.4740	046	0.3010	071	0.0990	096	0.0000
022	0.4670	047	0.2940	072	0.0900	097	0.0000
023	0.4600	048	0.2870	073	0.0810	098	0.0000
024	0.4530	049	0.2800	074	0.0720	099	0.0000
025	0.4470	050	0.2720	075	0.0640	100	0.0000

Table I. Count Rate Meter (CRM) Calibration. (Cont.)

300 III Scale

CRM Reading :	Density : gm/cu-cm :	CRM Reading :	Density : gm/cu-cm :	CRM Reading :	Density : gm/cu-cm :	CRM Reading :	Density : gm/cu-cm :
001	0.3380	026	0.1870	051	0.0200	076	—
002	0.3310	027	0.1800	052	0.0140	077	—
003	0.3260	028	0.1730	053	0.0060	078	—
004	0.3200	029	0.1670	054	0.0000	079	—
005	0.3150	030	0.1610	055	—	080	—
006	0.3090	031	0.1540	056	—	081	—
007	0.3030	032	0.1470	057	—	082	—
008	0.2970	033	0.1400	058	—	083	—
009	0.2910	034	0.1350	059	—	084	—
010	0.2850	035	0.1280	060	—	085	—
011	0.2790	036	0.1210	061	—	086	—
012	0.2730	037	0.1150	062	—	087	—
013	0.2670	038	0.1090	063	—	088	—
014	0.2600	039	0.1020	064	—	089	—
015	0.2540	040	0.0950	065	—	090	—
016	0.2490	041	0.0880	066	—	091	—
017	0.2430	042	0.0820	067	—	092	—
018	0.2370	043	0.0760	068	—	093	—
019	0.2310	044	0.0690	069	—	094	—
020	0.2250	045	0.0620	070	—	095	—
021	0.2180	046	0.0550	071	—	096	—
022	0.2120	047	0.0490	072	—	097	—
023	0.2060	048	0.0410	073	—	098	—
024	0.2000	049	0.0340	074	—	099	—
025	0.1930	050	0.0270	075	—	100	—

300 IIa scale is for B. V. of 12100--12220 v

300 III scale is for B. V. of 13100--13330 v

Fig. 6 and Fig. 7 are two typical tracings of the CRM recorder on 300 I scale and 300 III scale respectively.

Rotameter Calibration. The linear flow velocity of the air was measured by a rotameter of Brooks Rotameter Co. R-9M-25-2. Two floats and one pressure gauge were used to cover the range of flow rates used in this experiment. The aluminum float was used to measure the low flow rates, and the steel float was used to measure high flow rates. The rotameter was further calibrated by an Anemotherm (No. 1005 Mechanical Engineering, K.S.U.) for the high flow rates. For calibration in the low range, a wet test meter (WMT 4, Dept. of Chemical Engineering) was used. Five pressures of 5, 10, 15, 20, 25 psig were used as parameters. The calibration curve is presented in Fig. 8.

Procedure

The instruments were turned on at least 4 hours, usually overnight, before a run was made. The calibration was checked to make sure that the instrumentations were stabilized. The column was charged with the desired particles. Pressure drop data were taken, increasing the air velocity from below the minimum fluidizing velocity to about 90 ft/min. The air velocity was then decreased slowly until the air velocity was zero. The static bed height was then read. The air was then set at the desired velocity. The data were taken with 300 multiplier on 30-second time constants and with the bucking voltage appropriate for the particular density range. Data were generally taken at every half inch in the bed, starting 1 inch above the distributor.

Near the top of the fluidized bed, data were taken every quarter inch. The attenuation readings from the recorder chart were converted to average line density by the calibration table (Table I), plotted on rectangular graphs to obtain the density profile, and then the consistency test performed.

Consistency Test

To prove that the radioactive technique used for measuring the axial solid density was satisfactory and reproducible, the consistency test for several sets of experimental data was performed. The test was based on the principle of the conservation of mass, i.e., the actual mass of particles placed in the column should be equal to the integrated area of the solid-density profile.

For most runs the results obtained by this consistency test were within the allowable error. The procedure and the calculation method of this consistency test are as follows:

The area under the curve shown in Fig. 9 was graphically integrated by means of a planimeter. The total mass of particles determined in this manner is:

$$M^i = \frac{\text{(integrated area)} \times \text{(cross sectional area of column)}}{\text{(unit conversion constant)}}$$

If the actual mass of particles in the column is M , then the deviational error is:

$$\% \text{ error} = \frac{M^i - M}{M} \times 100\%$$

Table 2 summarizes the results of this consistency test. The positive or negative error for each run is listed.

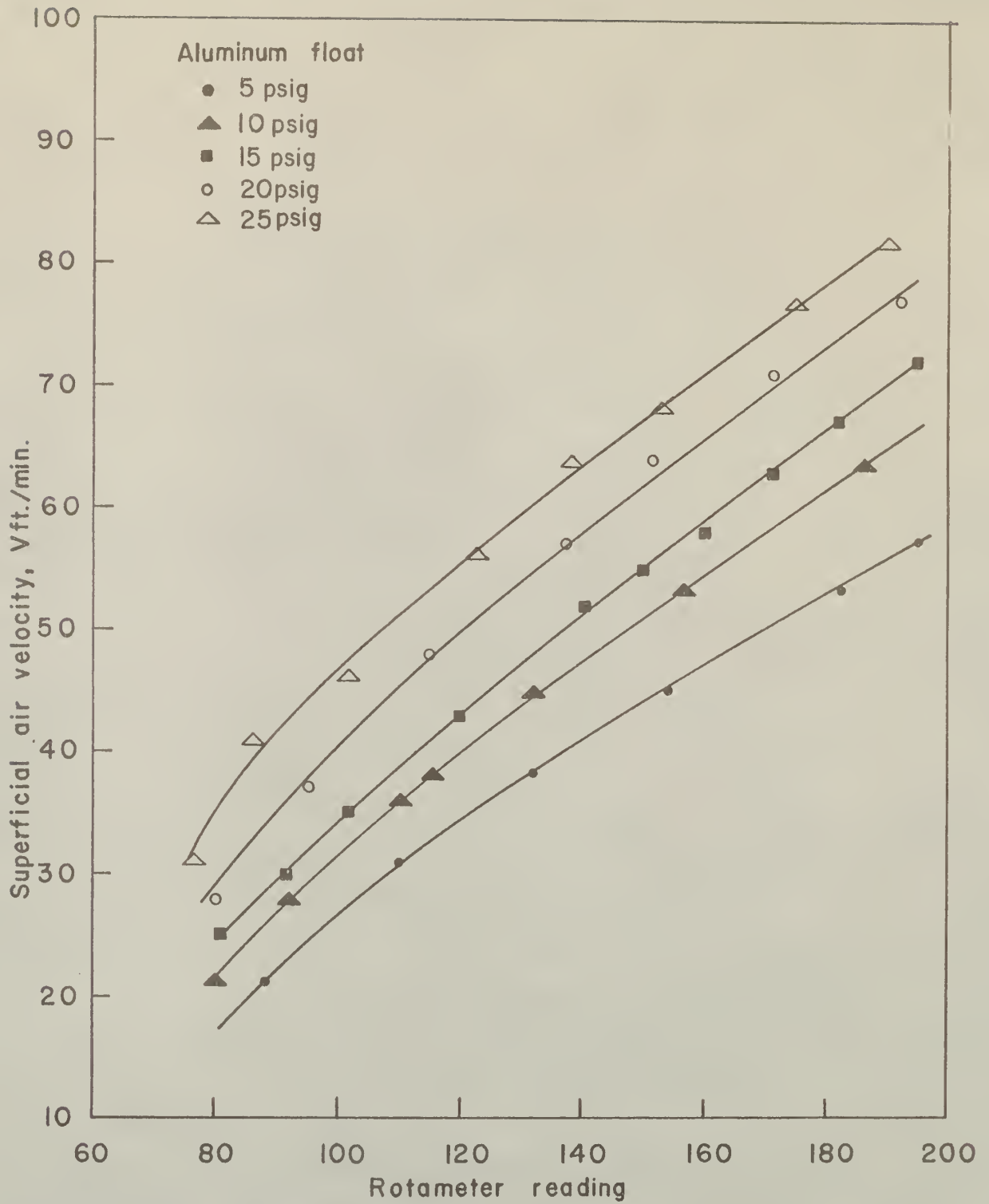


Fig. 8 Rotameter calibration

The factors which caused the error of this consistency test may be attributed to:

(a) The assumption that the average line density of solid is uniform across the cross sectional area at each axial position is not exactly valid since the density near the wall of column could be different from that in the middle of the column.

(b) There was an error due to radioactive decay or noise and other effects on the detecting power of the scintillation probe for γ -ray attenuation.

(c) When fluidizing, the particles in the bed were actually in a state of dynamic equilibrium. The line density detected is an average within the time constant used for the scintillation counting probe. Because of the complexity of the inter-particle disturbance, the speed of the particles at different locations in a certain layer could not be the same, but the time constant used for the scintillation counting probe was the same throughout. Therefore it could hardly be possible to take an exact average solid density which was consistent at any cross-section of the bed.

(d) The graphical integration must have introduced some error.

Fig. 10 shows the positive or negative percent error of this consistency test as a function of air velocity. The negative error consistently increased as the flow rate of air increased. At low flow rate of air, the error was almost negligible.

The fact that a larger error occurred at higher air velocity is logical since the particles were moving more vigorously and randomly in the bed. It might also be due to elutriation of small particles from the open top of the fluidizing column.

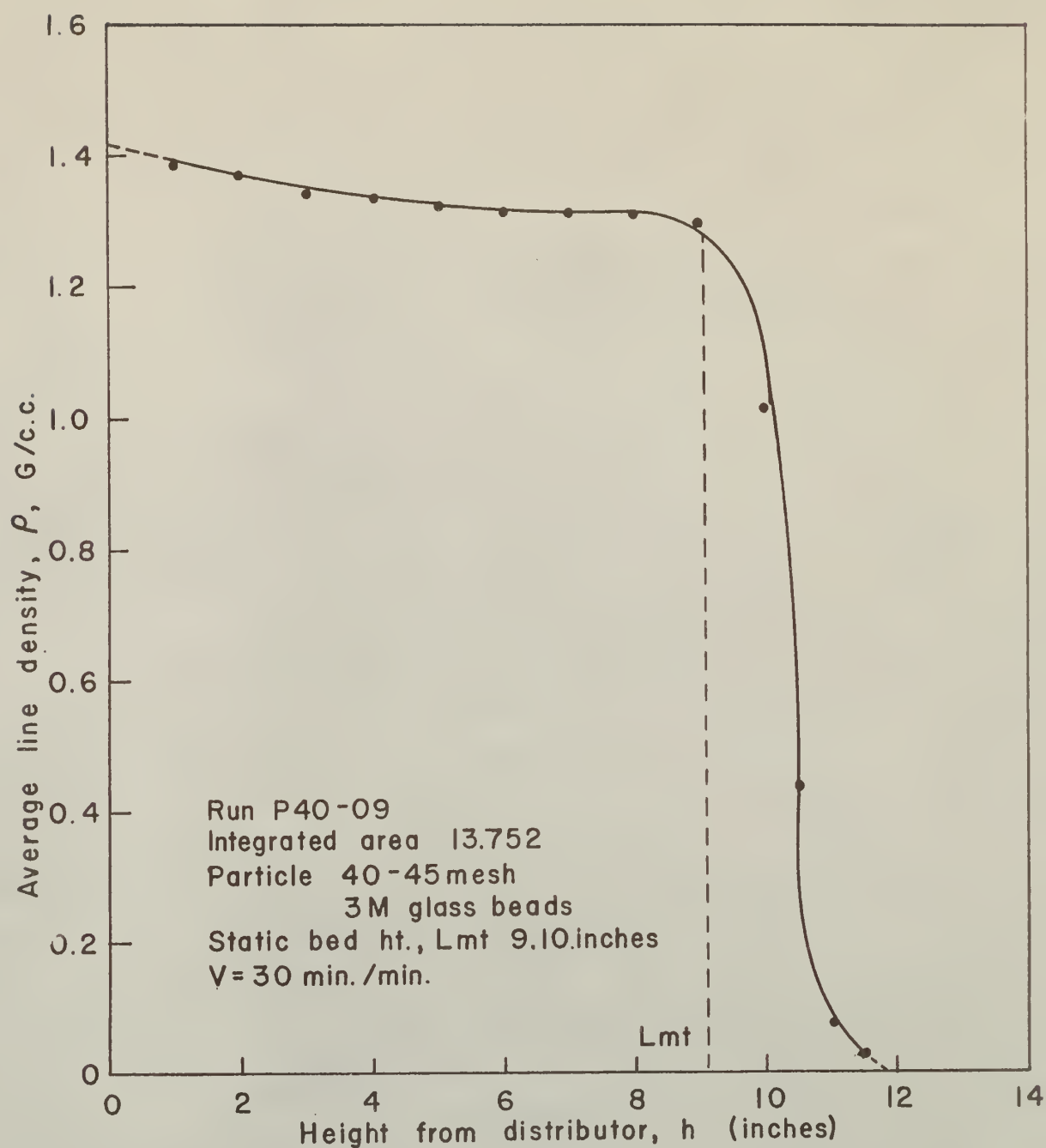


Fig. 9-(a) Axial density profile of Run P40-09 for graphical integration.

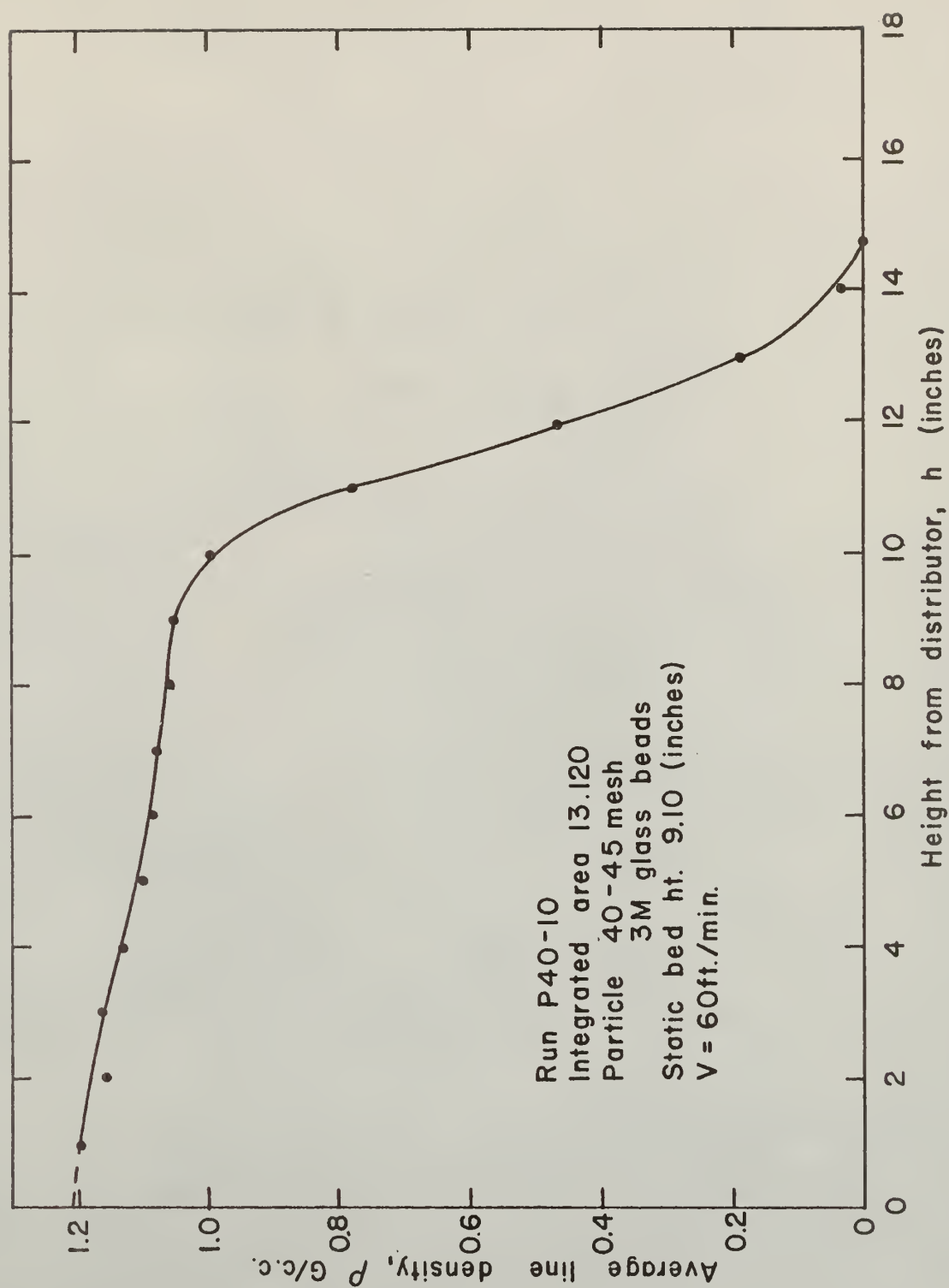


Fig. 9-(b). Axial density profile of Run P40-10 for graphical integration.

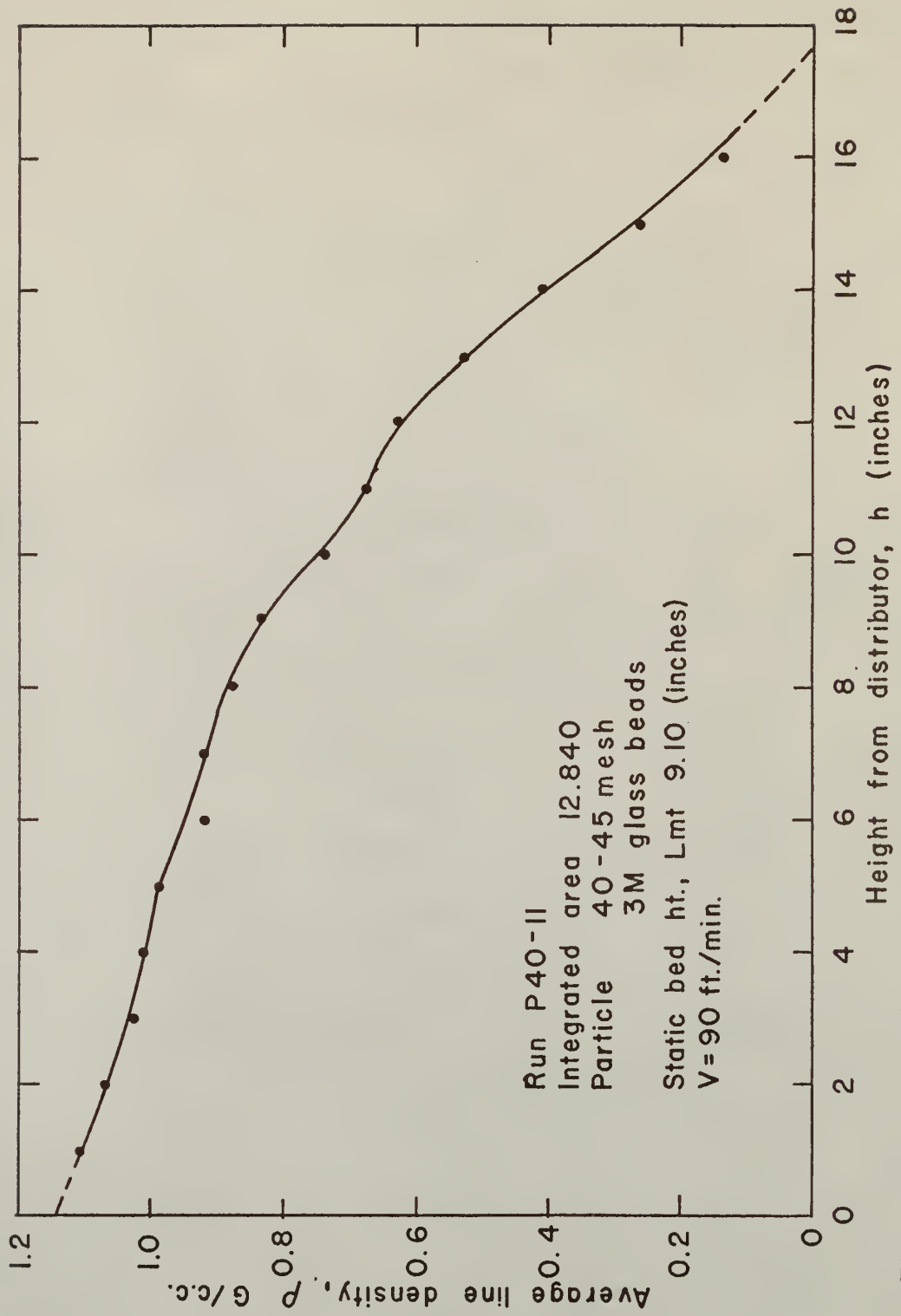


Fig. 9-(c) Axial density profile of Run P40-II for graphical integration.

Table 2. Consistency Test.

Run No.	St. Bed : Ht. in.	Air Vel. ft./min.	Bed Wt. : gram	Integrated Area		Bd. Wt. : Integ'd	Error, %	
				Below L _{mf}	Above L _{mf}	Total	+	-
P40-05	6.20	30	1893.803	8.168	1.196	9.364	1904.295	0.55
P40-06	6.20	60	1893.803	7.872	1.352	9.224	1875.824	0.95
P40-07	6.20	90	1893.803	6.120	1.768	8.288	1685.476	11.00
P40-01	2.75	30	839.993	3.780	0.392	4.172	848.432	1.00
P40-02	2.75	60	839.993	3.342	0.960	4.302	874.869	4.15
P40-03	2.75	90	839.993	2.890	1.184	4.074	828.503	1.370
P40-04	2.75	110	839.993	2.780	1.360	4.140	841.925	0.23
P40-09	9.20	30	2810.158	12.100	1.612	13.752	2796.654	0.480
P40-10	9.20	60	2810.158	10.044	3.076	13.120	2668.128	4.010
P40-11	9.20	90	2810.158	9.008	3.832	12.840	2611.180	6.060
P80-04	6.10	30	1812.273	6.944	2.012	8.954	1821.323	0.50
P80-05	6.10	60	1812.273	5.988	2.388	8.376	1703.372	5.230
P80-06	6.10	90	1812.273	5.328	2.656	8.024	1631.788	8.390
P80-07	8.75	30	2599.573	9.720	2.664	12.384	2518.453	3.120
P80-08	8.75	60	2599.573	8.560	2.952	11.512	2341.120	9.940
P80-10	3.15	30	935.846	3.640	1.288	4.928	991.597	5.96
P80-11	3.15	60	935.846	3.190	1.446	4.636	942.793	0.74

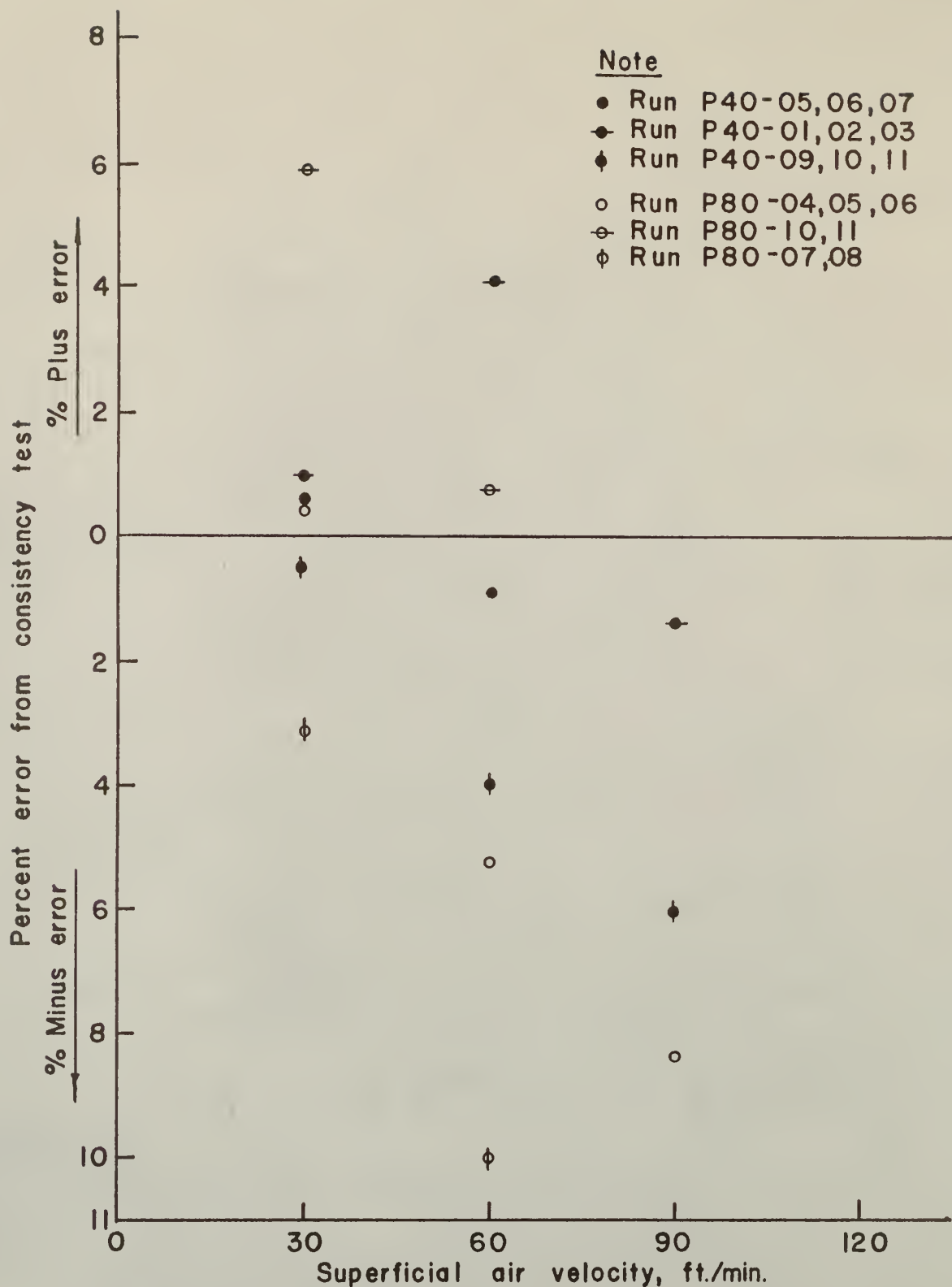


Fig. 10 Error in consistency test as a function of air velocity.

CHARACTERISTIC AXIAL DENSITY DISTRIBUTION OF GAS-SOLID, AGGREGATIVE FLUIDIZED BEDS

General Observations

Typical characteristic axial solid density profiles of a gas-solid fluidized bed are shown in Fig. 11 and Fig. 12. Similar profiles were obtained for all experimental runs for different static bed heights and under different air velocities. Even the beds mixed particles yielded similar profiles. Two distinct zones can be found in this typical distribution curve: in the range AB of Fig. 11, the solid density stays fairly constant. This is called the constant density zone. After point B the solid density decreases sharply as the distance from the bottom of the bed increases. This portion of the profile is called the falling density zone.

These typical density profiles show qualitatively the properties of gas-solid fluidized beds under the effect of several operational variables: --

Dotson (5) studied the effects of five primary variables on the uniformity of fluidization in a 4-inch diameter column by a statistically designed experiment. He measured the density fluctuations inside the bed by a capacitance method and concluded that among the five variables, i.e., distributor, bed height, particle size, probe height and gas velocity, gas velocity had by far the greatest effect on the uniformity, with uniformity generally decreasing with an increasing gas velocity.

In the present study with the γ -ray technique, the capacitance probe, which did effect the flow pattern in a fluidized bed, was not present. Thus its effect was eliminated. Only one type of gas distributor (canvas filter cloth) was used in all the experimental runs. Therefore, only three

operational variables were studied. The effect of these three operational variables on the axial solid density distribution will be discussed.

Effect of Air Velocity

Fig. 11 shows the effect of air velocity upon the fluidized bulk density profiles. It is shown that the density in the constant density zone decreased with increasing air velocity, but the slope of the falling density zone becomes smaller as the air velocity increases. This was expected since more air was passing through the bed at higher air velocity and, therefore, the bed was expanded.

Fig. 11 also indicates that this two-zone profile becomes less apparent as the air flow rate increases. At flow rate over 90 ft/min., the line average density falls from the bottom of the bed to the top layer of the fluidizing bed. In other words, when the air velocity is low the constant density zone clearly exists and the bed is closer to the ideally expanded fixed bed. If the air velocity is high, the air which carries the particles becomes more turbulent and the particles are in more vigorous and randomized movement. Under these conditions there is no section in the whole bed that has a uniform solid density.

Effect of Static Bed Height

Figs. 13 (a-d) are the dimensionless axial density profiles, in which ρ/ρ_{pa} was plotted against h/h_{pa} . It appears from these graphs that the static bed effect on the density profile is relatively small. The dimensionless density profiles for different static bed heights fluidizing at the same

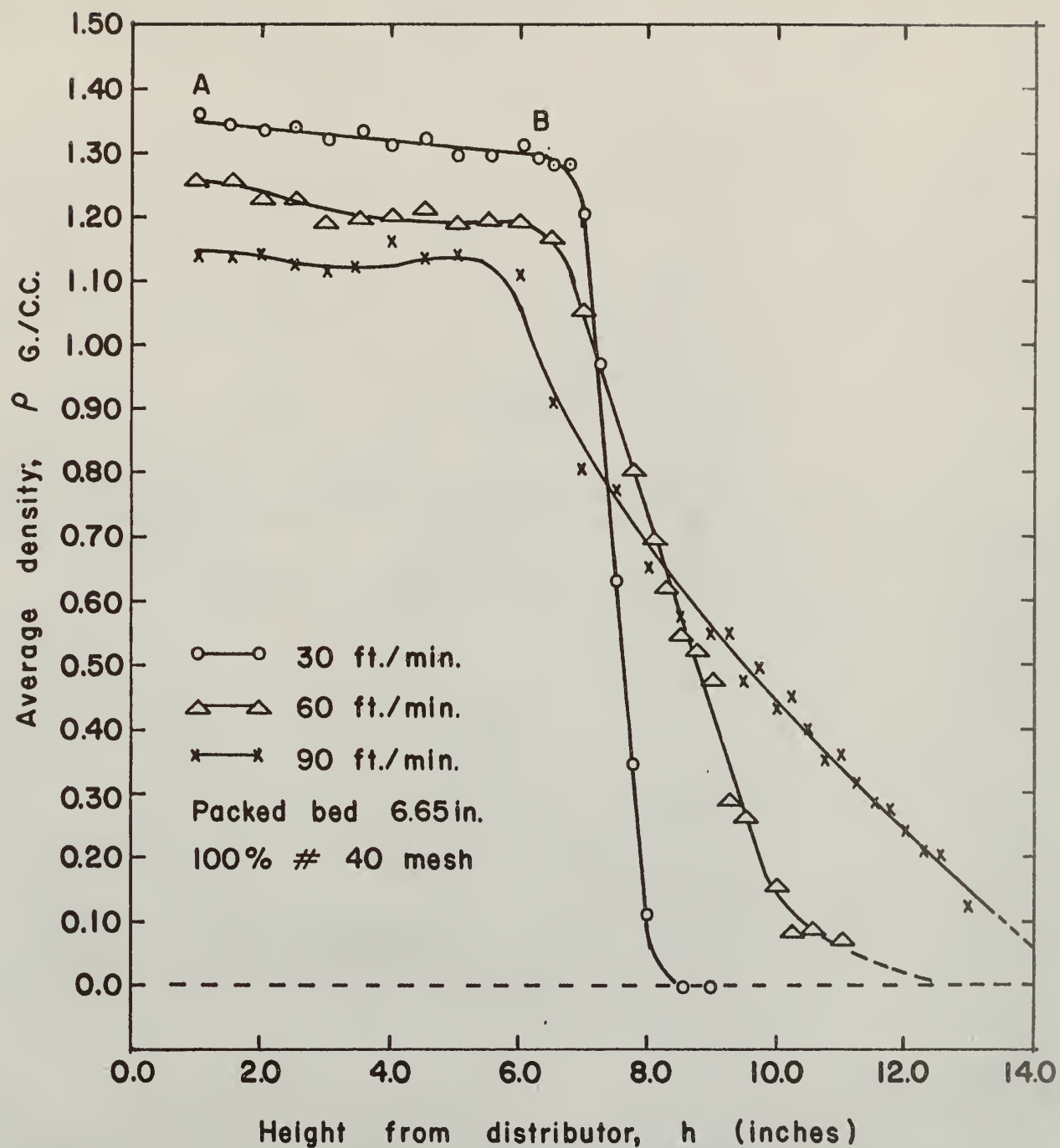


Fig. II. Effect of air velocity upon the density profiles.

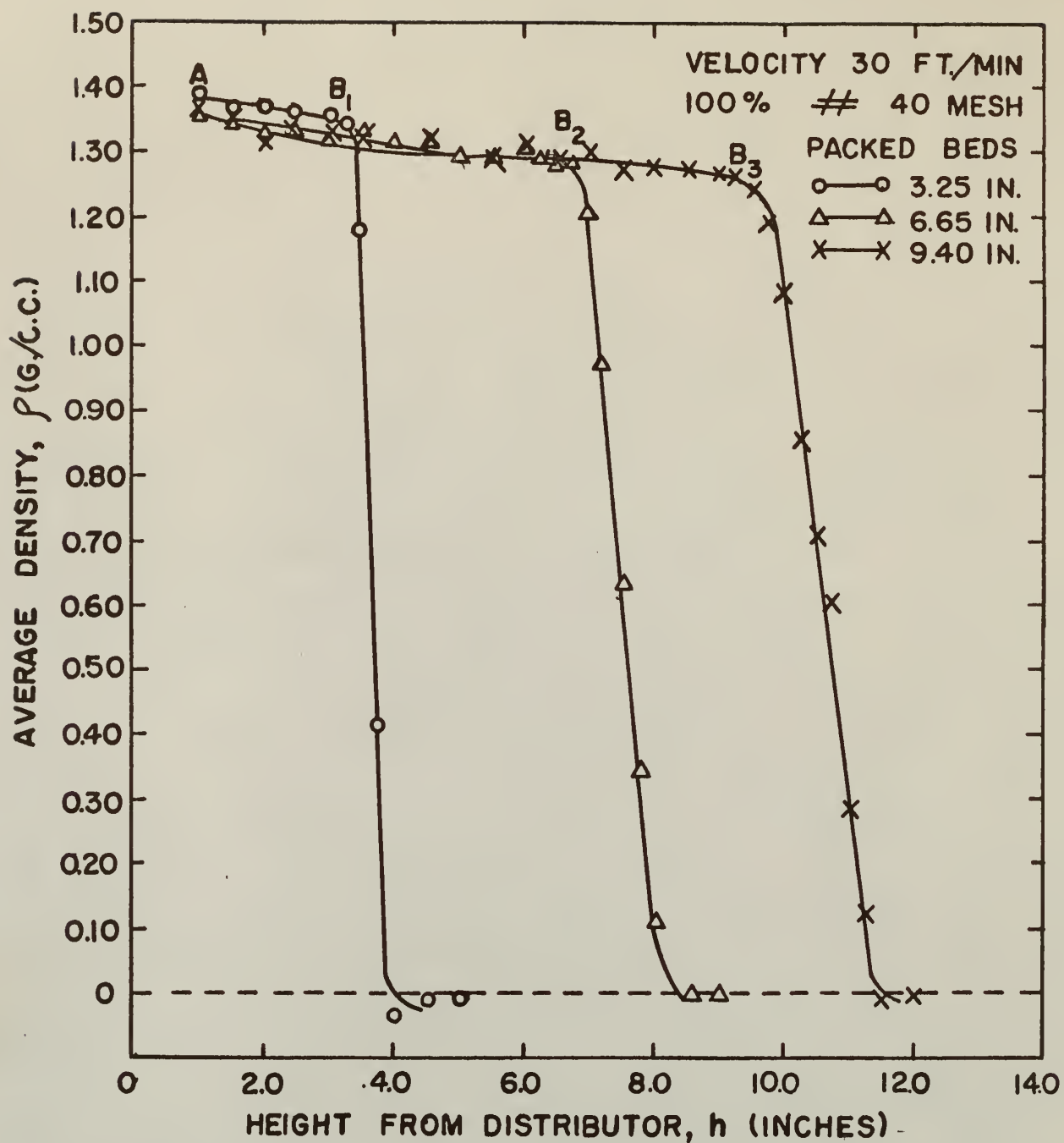


Fig. 12. Effect of static bed height upon density profiles.

air velocity can almost be represented by one curve. This is especially true at low fluidizing air velocities. The deviations between such curves becomes greater as the air velocity increases. The deviation from the common dimensionless density profile becomes more significant for the individual profile corresponding to the smallest bed height. This is shown in Figs. 13 (c) and 13 (d). This result may be explained as follows:

The ratio of the static bed height to the column diameter instead of the static bed height itself is probably the significant variable affecting the bed expansion and the density profile. This is the proper combination of entrance and wall effects. When the bed is shallow and the ratio is small, the air passing through could hardly have time to form well-shaped bubbles before it leaves the fluidizing bed. The two phases of fluidization for the shallow bed are therefore not as pronounced as in the case of deep static bed.

It may be concluded that at higher air velocity, the effect of the static bed height on the density profile becomes more noticeable when the ratio of static bed height to the column diameter is less than one (the diameter of fluidizing column used was 3.9765 inch). If the ratio is greater than one, the effect of the static bed height on the dimensionless density profile is negligible. This is consistent with the observation made by Gomezplata et.al.,(8). They studied the entrance effect on the conversion of the catalytic cracking of cumene. Their results are shown in Fig. 14 to compare with and to substantiate the present observation.

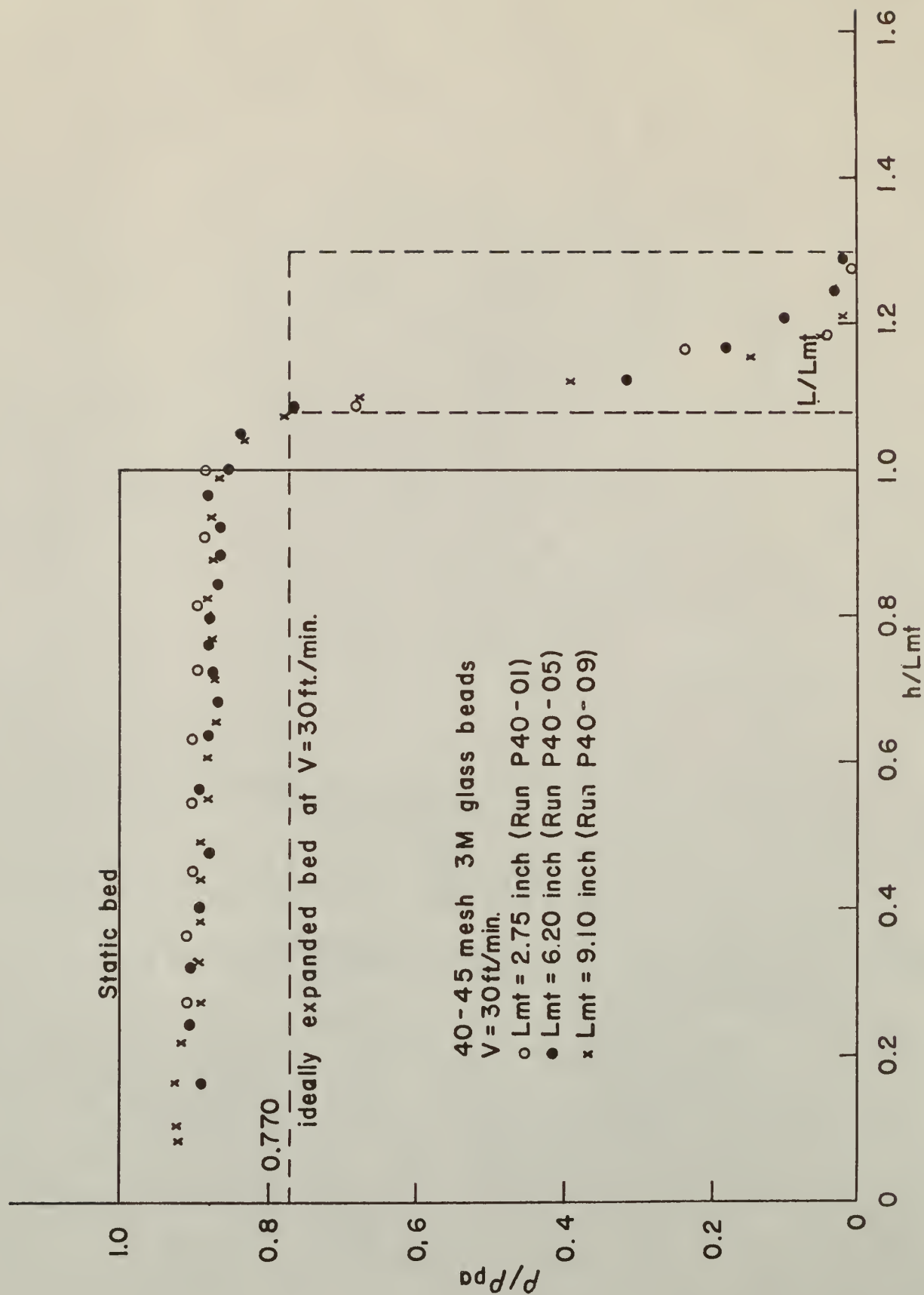


Fig. 13-(a). Dimensionless density profile at $V=30\text{ft./min}$ for various static bed heights.

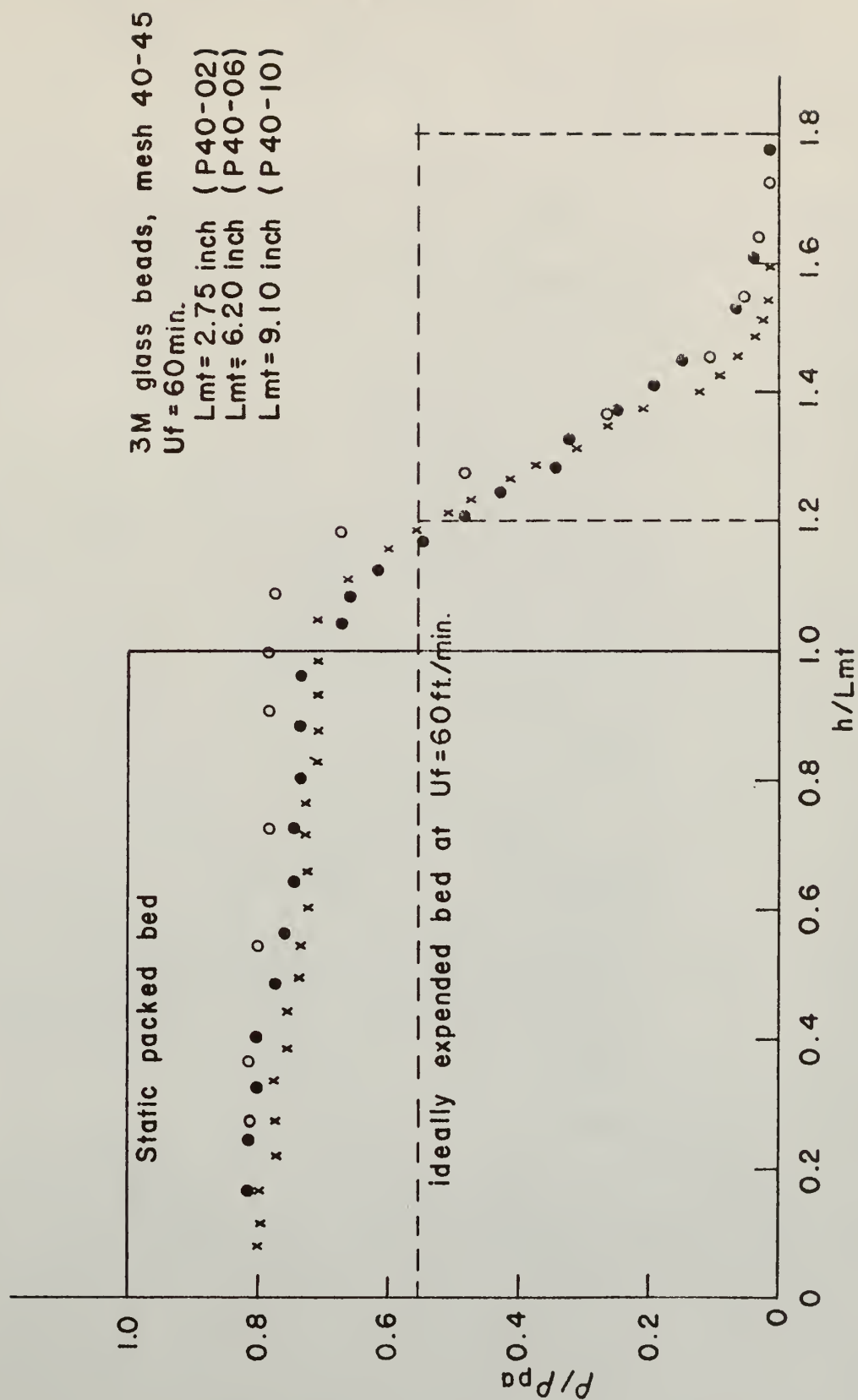


Fig. 13-(b). Dimensionless density profile at $V = 60 \text{ ft./min.}$ for various static bed height.

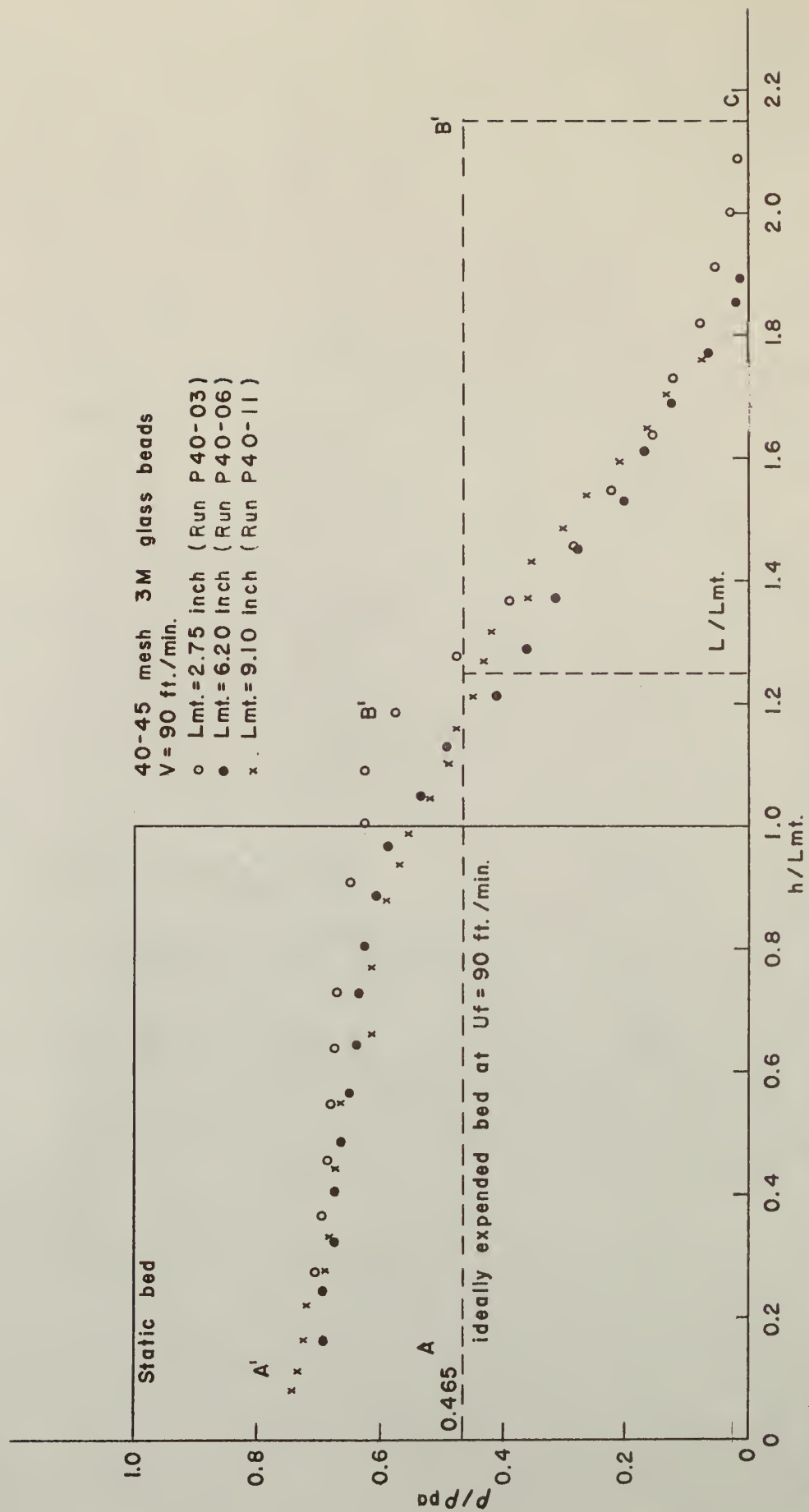


Fig. 13-(C) Dimensionless density profile at $V = 90$ ft./min. for various static bed heights.

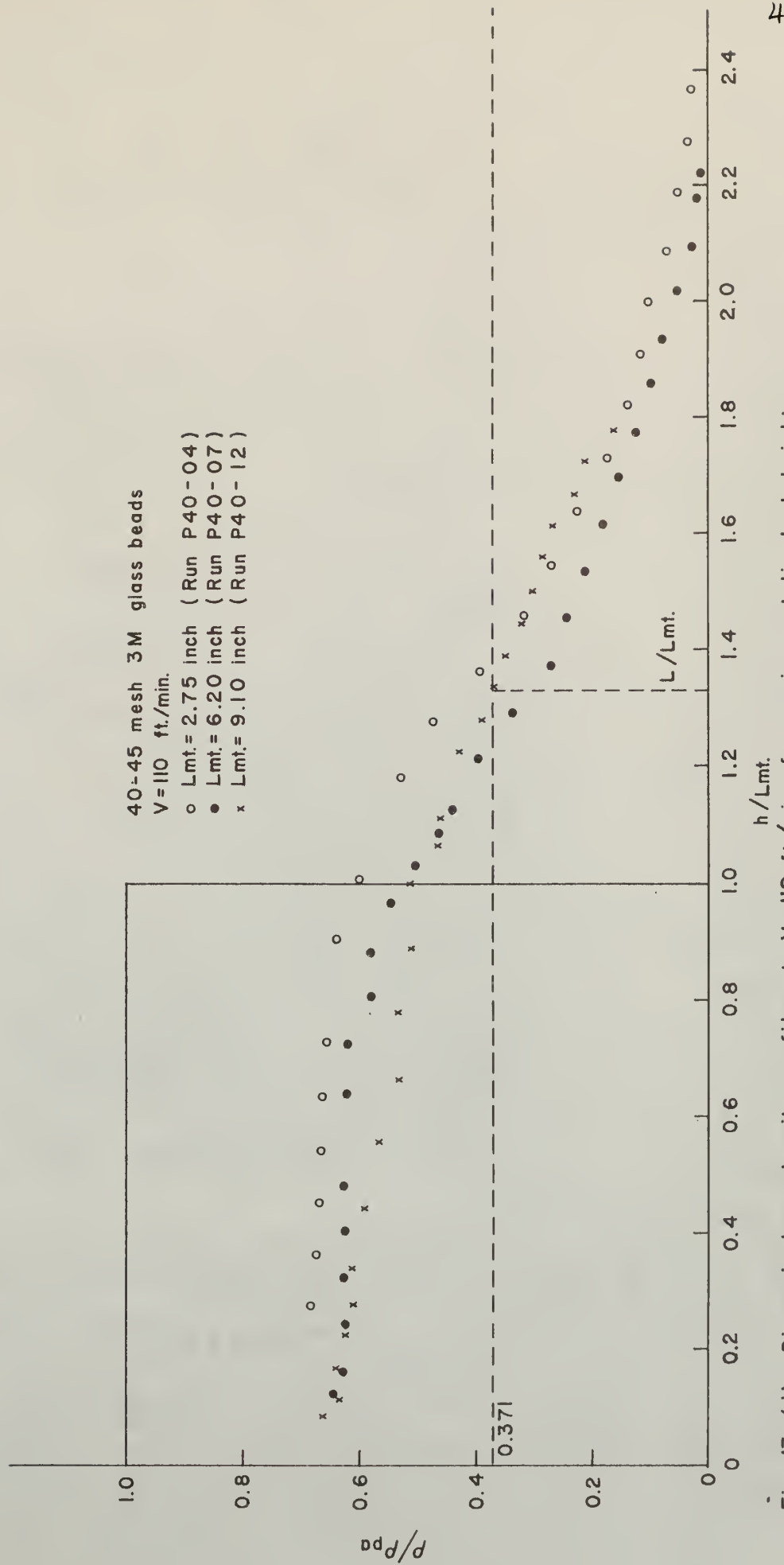


Fig. 13-(d) Dimensionless density profile at $V = 110$ ft./min. for various static bed heights.

Effect of Particle Size and Bed Composition

The effect of particle size and bed composition could not be determined either qualitatively or quantitatively without defining some index for aggregative fluidization. Stewart (25) has discussed the particle size effect with a defined ISU index. His results are consistent with the findings of Dotson (5) and Baumgarten (3).

More details and quantitative discussion of the effects of operational variables on aggregative fluidization will be found in later sections.

ANALYSIS OF DATA AND CORRELATIONS

As mentioned in the previous section, the axial density profiles may be divided into a constant density zone and a falling density zone. The correlation of the density profile as a function of the operational variables will be obtained for the two zones separately. It was usually considered (1) that the constant density zone extended from the bottom of the bed to the position corresponding to the static bed height, and the falling density zone existed above the static bed height.

Bed Density Correlation in the Constant-Density Zone

The density in the constant-density zone is fairly constant, though a considerable density fluctuation may be observed from the density profile at high air velocities. From the qualitative observation made in the previous

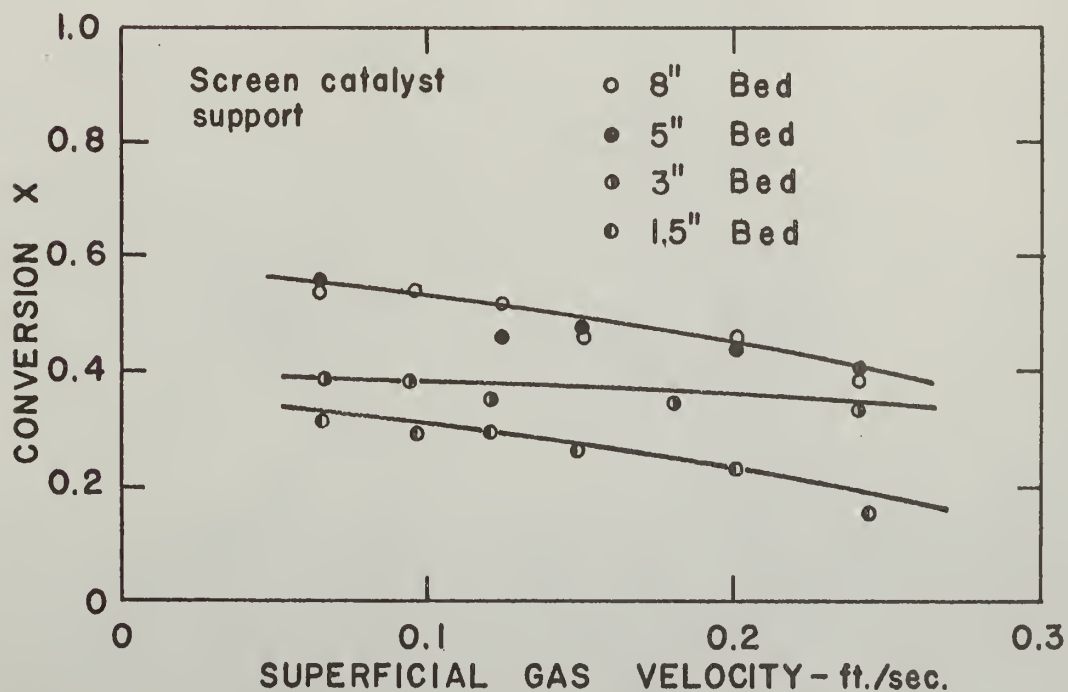
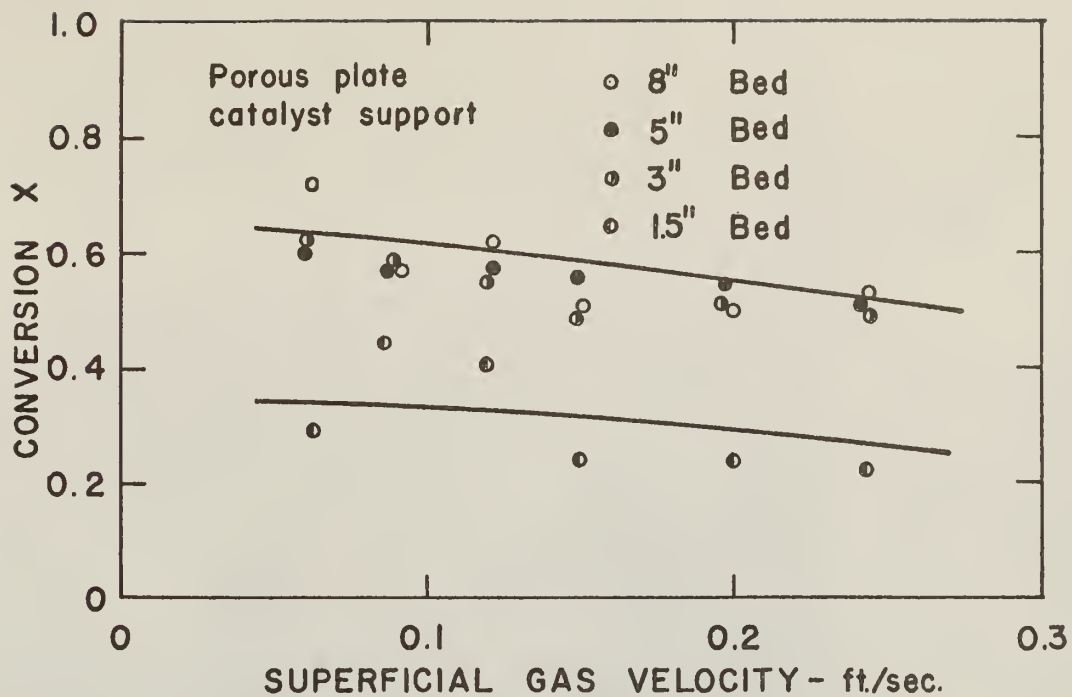


Fig. 14. Entrance effect on the conversion of cumene. [Gomezplata & Shuster (8)]

section, we know that air velocity is the major factor affecting the shape of the density profile, whereas the static bed height has little effect. Thus the density in the constant-density zone can be correlated as a function of air velocity and particle diameter.

Let $\bar{\rho}_d$ represent the average density in the constant density zone (where the region ranges from the bottom of the column to the position corresponding to the static bed height, L_{mf}); M_{s2} , the weight of particles fluidizing below L_{mf} ; and A , the cross-sectional area of the fluidizing column, then:

$$\bar{\rho}_d = \frac{M_{s2}}{A L_{mf}} \quad (5)$$

The particles blown off and suspended above L_{mf} , are then

$$M = M_{s1} + M_{s2} \quad (6)$$

where M is the total weight of particles charged in the column. It was mentioned by Bakker et.al.(2) that the volume of the particles suspended in the space above L_{mf} , i.e. Vol_{s1} , corresponds to the product of the volume of the gas bubbles appearing below L_{mf} , and the concentration in the prefluidized state. Thus,

$$Vol_{s1} = (1 - \epsilon_{mf}^*) Vol_b \quad (7)$$

If we consider that A' is the part of area A which is occupied by the dense phase emulsion, then $(A - A')$ is the area crossed by the bubbles. The superficial velocity of bubbles, as indicated by Baumgarten et.al.(3), is a function of bubble size and is different at each layer of the fluidized bed. He observed that the size of a bubble grows as it moves away from the

* ϵ_{mf} , is the void space of bed at minimum fluidizing state.

distributor. But to avoid this complexity, we can assume that there is a mean bubble velocity, \bar{V}_b , which is constant throughout the section below L_{mf} . With this assumption:

$$G = G_b + G_{mf} \quad (8)$$

where G is the volumetric air velocity, G_b is the volumetric bubble velocity, and G_{mf} , the volumetric velocity of minimum fluidization.

Therefore, from equation (8):

$$VA = \bar{V}_b (A - A') + V_{mf} A' \quad (8')$$

Rearranging equation (8') we get:

$$A' = \frac{A (\bar{V}_b - V)}{\bar{V}_b - V_{mf}} \quad (9)$$

Let t_b be the residence time of bubbles in the bed below L_{mf} . Then:

$$t_b = \frac{L_{mf}}{\bar{V}_b} \quad (10)$$

The volume occupied by bubbles below L_{mf} is:

$$Vol_b = G_b t_b = \bar{V}_b (A - A') \frac{L_{mf}}{\bar{V}_b} \quad (11)$$

Substituting equation (9) into Equation (11) gives:

$$\begin{aligned} Vol_b &= L_{mf} \left[A - \frac{A(\bar{V}_b - V)}{(\bar{V}_b - V_{mf})} \right] \\ &= \frac{AL_{mf} (V - V_{mf})}{\bar{V}_b - V_{mf}} \end{aligned} \quad (12)$$

From Equations (7) and (12), the volume of the solid particles above L_{mf} is obtained.

$$Vol_{s_1} = \frac{AL_{mf} (V - V_{mf})}{\bar{V}_b - V_{mf}} (1 - \epsilon_{mf}) \quad (13)$$

then,

$$M_{s_1} = \rho_s Vol_{s_1} = \rho_s (1 - \epsilon_{mf}) \frac{AL_{mf} (V - V_{mf})}{\bar{V}_b - V_{mf}} \quad (14)$$

and

$$M_{s_2} = M - \frac{\rho_s (1 - \epsilon_{mf}) AL_{mf} (V - V_{mf})}{\bar{V}_b - V_{mf}} \quad (15)$$

Substitution of Equation (15) to Equation (5) leads to:

$$\bar{\rho}_d = \frac{M_{s_2}}{AL_{mf}} = \frac{M}{AL_{mf}} - \frac{\rho_s (1 - \epsilon_{mf})}{(\bar{V}_b - V_{mf})} (V - V_{mf}) \quad (16)$$

$$\bar{\rho}_d = \rho_{mf} - \frac{\rho_s (1 - \epsilon_{mf})}{(\bar{V}_b - V_{mf})} (V - V_{mf})$$

since,

$$1 - \epsilon_{mf} \doteq \frac{\rho_{mf}}{\rho_s}$$

then,

$$\begin{aligned} \bar{\rho}_d &= \rho_{mf} - \frac{\rho_{mf}}{(\bar{V}_b - V_{mf})} (V - V_{mf}) \\ &= \rho_{mf} - C(V - V_{mf}) \end{aligned} \quad (17)$$

where $C = \frac{\rho_{mf}}{(\bar{V}_b - V_{mf})}$

Since the average bubble velocity through the bed below L_{mf} has been assumed constant, C in equation (17) is a constant and the average density in the constant density zone has a linear relationship with the superficial air velocity.

The data plotted on Fig. 15 (a) and Fig. 15 (b) confirm this linear relationship between $\bar{\rho}_d$ and $(V - V_{mf})$ fairly well with the static bed height showing a negligible effect.

Using the least squares method, the intercepts and slopes of the straight lines from the experimental data were obtained. The empirical equations which relate $\bar{\rho}_d$ with $(V - V_{mf})$ for 40-45 mesh and 80-100 mesh 3M glass beads are as follows:

(i) For 40-45 mesh glass beads:

$$\bar{\rho}_d = 1.371 - 0.00536 (V - V_{mf}, 40 \text{ mesh})$$

(ii) For 80-100 mesh glass beads:

$$\bar{\rho}_d = 1.269 - 0.00439 (V - V_{mf}, 80 \text{ mesh})$$

where $\bar{\rho}_d$ is in gm/cu-cm

V, V_{mf} are in ft/min

$V_{mf}, 40 \text{ mesh}$ is the minimum velocity of fluidization of 40-45 mesh glass beads with air, which is 24.3 ft/min

$V_{mf}, 80 \text{ mesh}$ is the minimum velocity of fluidization of 80-100 mesh glass beads with air, which is 4.89 ft/min

Bed Density Correlation in the Falling Density Zone

The fluidized bulk density in this zone decreases as the height from distributor increases. In all cases, as has been shown in Fig. 13, the density distribution in this falling density zone may be represented by a probability function. For the experimental runs at low air velocity, the density distribution in this falling zone can be fitted almost exactly by the one-tailed normal distribution function, if we consider that the

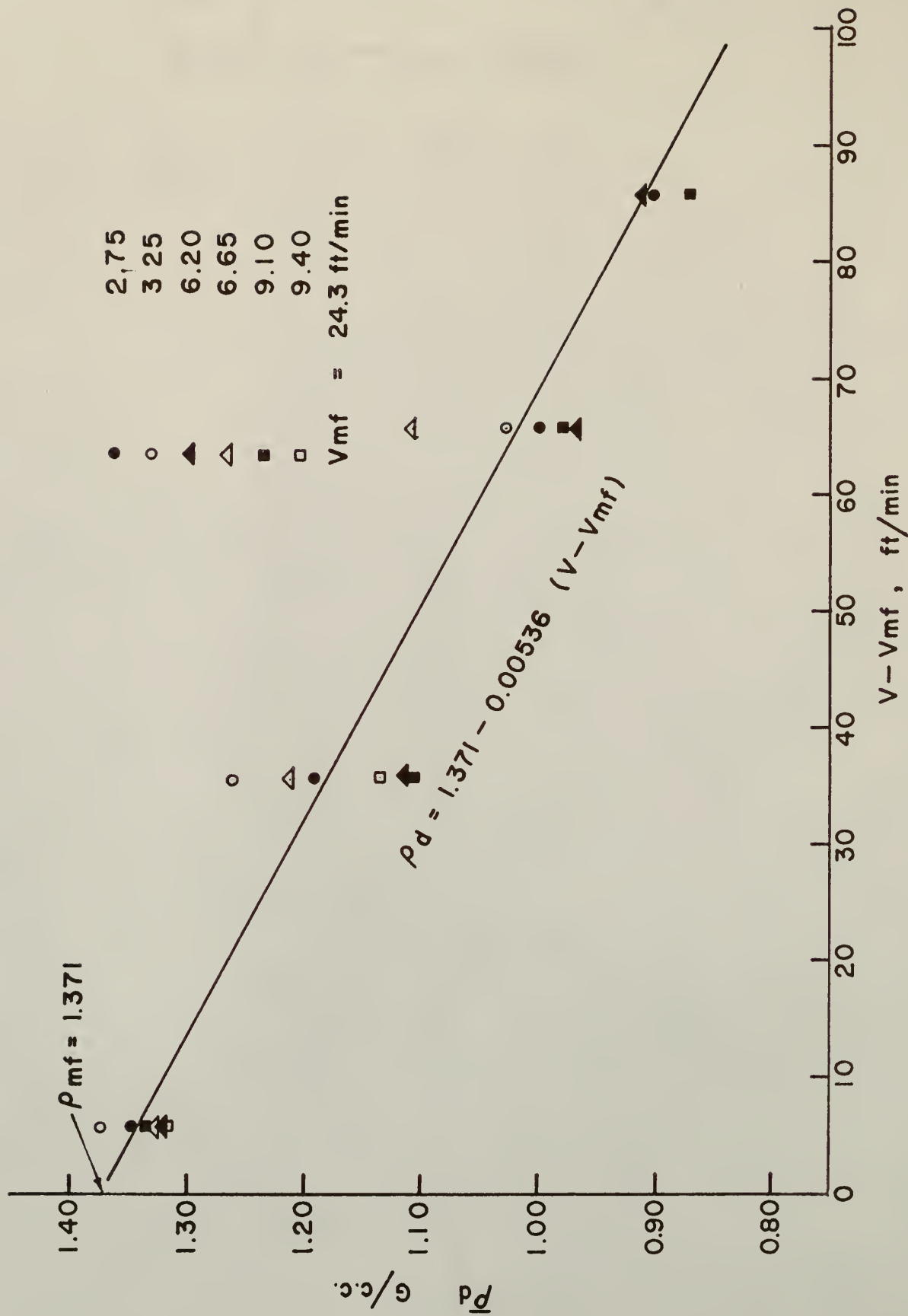


Fig. 15-a. Correlation for 40-45 mesh glass beads

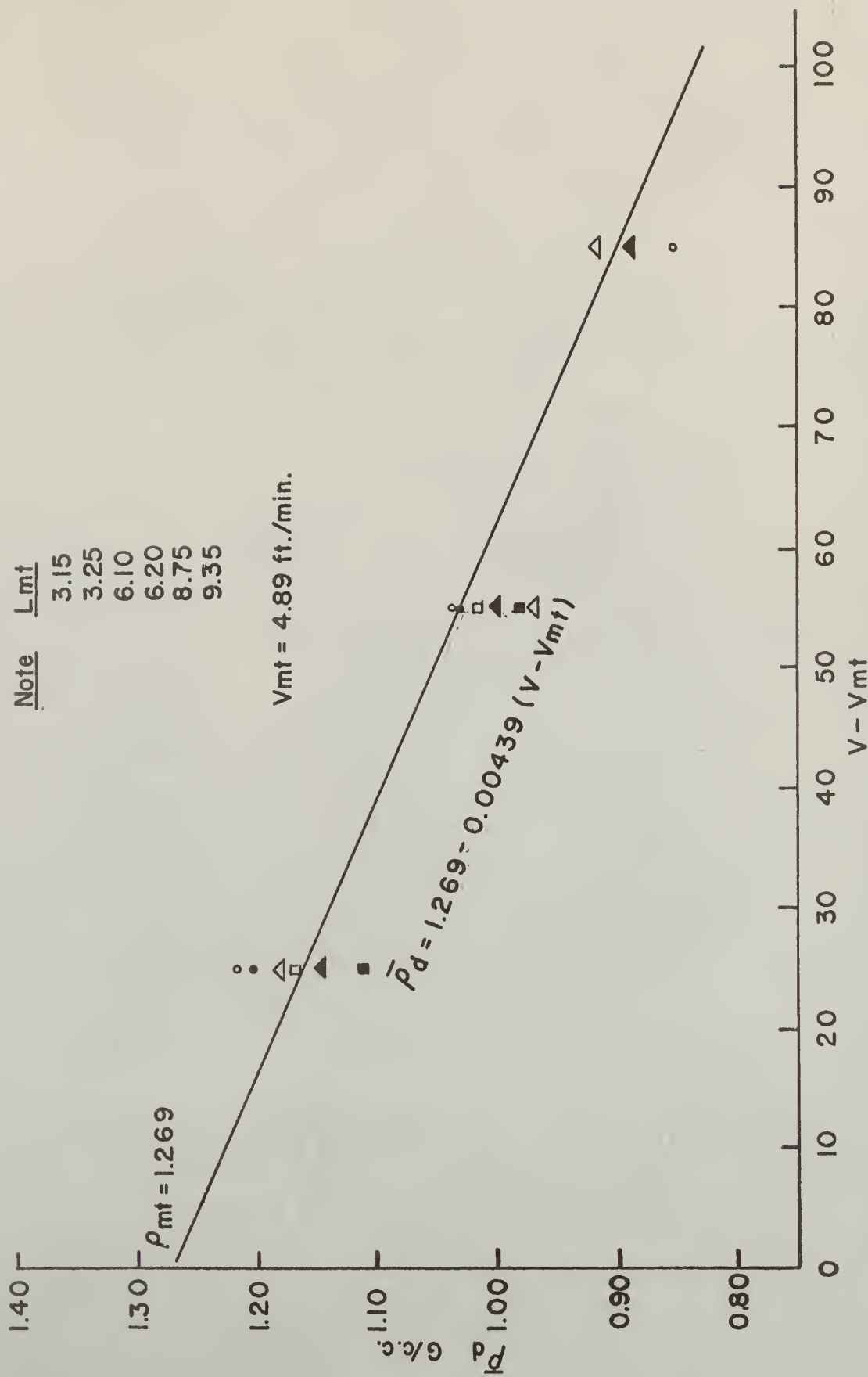


Fig. 15-(b). $\bar{\rho}_d$ Correlation for 80-100 mesh glass beads.

falling density zone starts at the location corresponding to the static bed height. But for the falling density distribution obtained from the bed fluidizing at higher air velocity, the profile becomes rather skewed. We can hardly find a simple form of probability equation to fit this profile. Consequently, it is not an easy task to find a unique and general form of probability equation capable of describing the falling density profiles for all ranges of air velocity.

Thus, it becomes necessary to search for an empirical equation which, with some coefficients determined from the experimental data, will describe quantitatively the state of aggregative fluidization. For instance, the varying slope of the falling density profile could be a simple index for this purpose.

It was found that a function in the form:

$$\rho/\rho_{pa} = A_0 \exp \left[B_0 (h/L_{mf} - 1)^2 \right] \quad (18)$$

would correlate fairly well the falling density profiles. At a height corresponding to L_{mf} (the position at prefluidized state) the fluidized bulk density is ρ_d , which can be obtained from the axial density profile. In most cases it was the same as $\bar{\rho}_d$, the average density in the constant density zone. Values of $\bar{\rho}_d$ have been plotted as function of air velocity and particle size in Fig. 15. The value of A_0 in equation (18) is then equal to ρ_d/ρ_{pa} , which can be calculated for each condition.

The problem now is to solve for B_0 , which indicates quantitatively the slope of the correlated density profile for the falling density zone. It may be considered as an index to describe the fluidizing states under various operating conditions.

By taking logarithms of both sides of equation (18), we get the following linear equation:

$$\begin{aligned}\log (\rho/\rho_{pa}) &= \log (\rho_d/\rho_{pa}) + B (h/L_{mf} - 1)^2 \\ &= A + B (h/L_{mf} - 1)^2\end{aligned}\quad (19)$$

It should be noted that equation (19) is only an empirical equation to fit the experimental falling density profiles. It is not fitted equally well by the experimental data obtained under different fluidizing conditions. The general trend is shown in Fig. 16.

For all sizes of glass beads fluidizing at lower air velocities, the falling density profiles were fitted better by the linear relation of equation (19); while for those fluidizing at higher air velocities, the linear relationship was not the best fit to the profile. Fig. 16 is the falling density profiles transformed into the form of equation (19). Even though the linear correlation of the dimensionless density to the dimensionless height was not as good for the runs fluidizing at higher air velocities as that at lower velocities, it is possible to obtain a best straight line by using the method of least squares. Thus the slope, B, and the intercept, $A = \log (\rho_d/\rho_{pa})$ can be found for all sets of experimental data fluidizing at various conditions. They are listed in Table 3 (Appendix).

The index B was plotted in Fig. 17 (a), 17 (b) and 17 (c) with the air velocity variable for three different sizes of glass beads. This index converges to a single value as the air velocity increases. It indicates that at high air velocity, the particles carried up by the air bubbles are uniformly dispersed in the falling density zone. No matter what the original static bed height, the value of index B is the same if fluidizing at high air velocity.

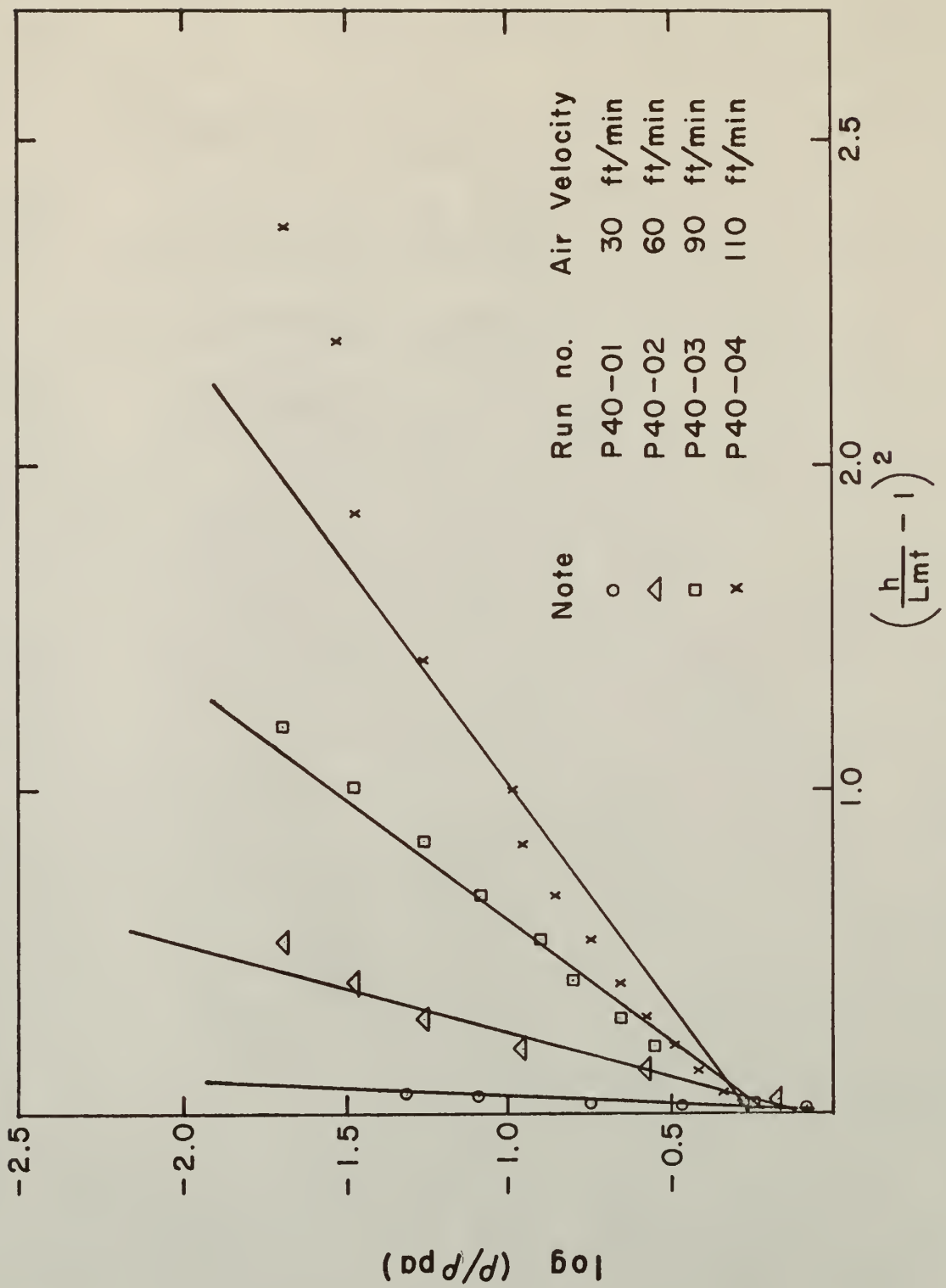


Fig. 16. Fitting experimental data to the empirical equation $\log (P/P_{pa}) = A + B \left(\frac{h}{Lmf} - 1\right)^2$

Fig. 18 further verifies that the effect of the static bed height on aggregative fluidization is pronounced only at low air velocity close to the minimum velocity of fluidization. In Fig. 19, the plot of index B vs. the particle size shows that the particle size affects the state of aggregative fluidization only at lower air velocities. In other words, the particles are distributed in a similar manner over the falling density zone (at high air velocity), no matter what size of particles the bed is composed of.

The value of A, which is actually the ratio of fluidized bulk density at the position of the static bed height to the bulk density of the same particles in their packed state, was correlated in a similar way as that done for index B. Fig. 20 shows that the index A is varying with air velocity for all static bed heights. In all cases (Fig. 20, a-c) the effect of static bed height on the index A is significant when $L_{mf}/D_c < 1$. When this ratio is greater than one, the fluidized bed expansion was essentially the same for all static bed heights fluidizing at the same air velocity. For the 80-100 mesh glass beads, the index A correlation with air velocity converges to one curve for all static bed height, even when L_{mf}/D_c is less than one. This leads to the conclusion that when the particles size is small, the entrance effect does not affect the index A (Fig. 20, c).

The separate effect of particle size on the index A is mostly due to the relative magnitude of the bulk density of different particles in the packed state (ρ_{pa}). The size effect is practically the same at all air-velocity levels as has been shown in Fig. 21.

The effect of bed composition on the values of index A and index B is shown in Fig. 22. The beds were originally at a static height of 6.5 inch and were composed of various fractions of 40-45 mesh and 80-100 mesh particles.

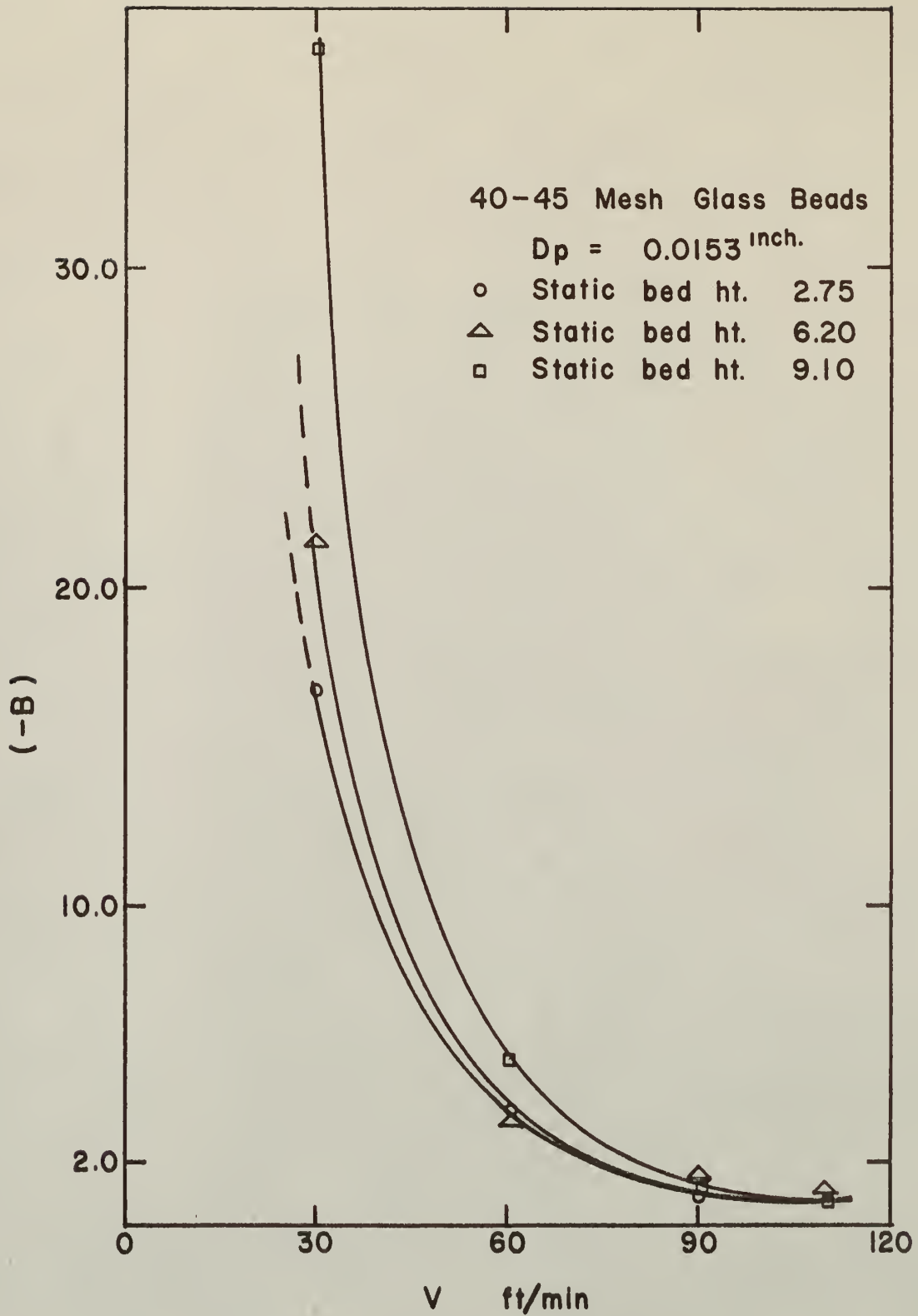


Fig. 17-a Correlation of Index-B v.s. air velocity with static bed height as parameter.

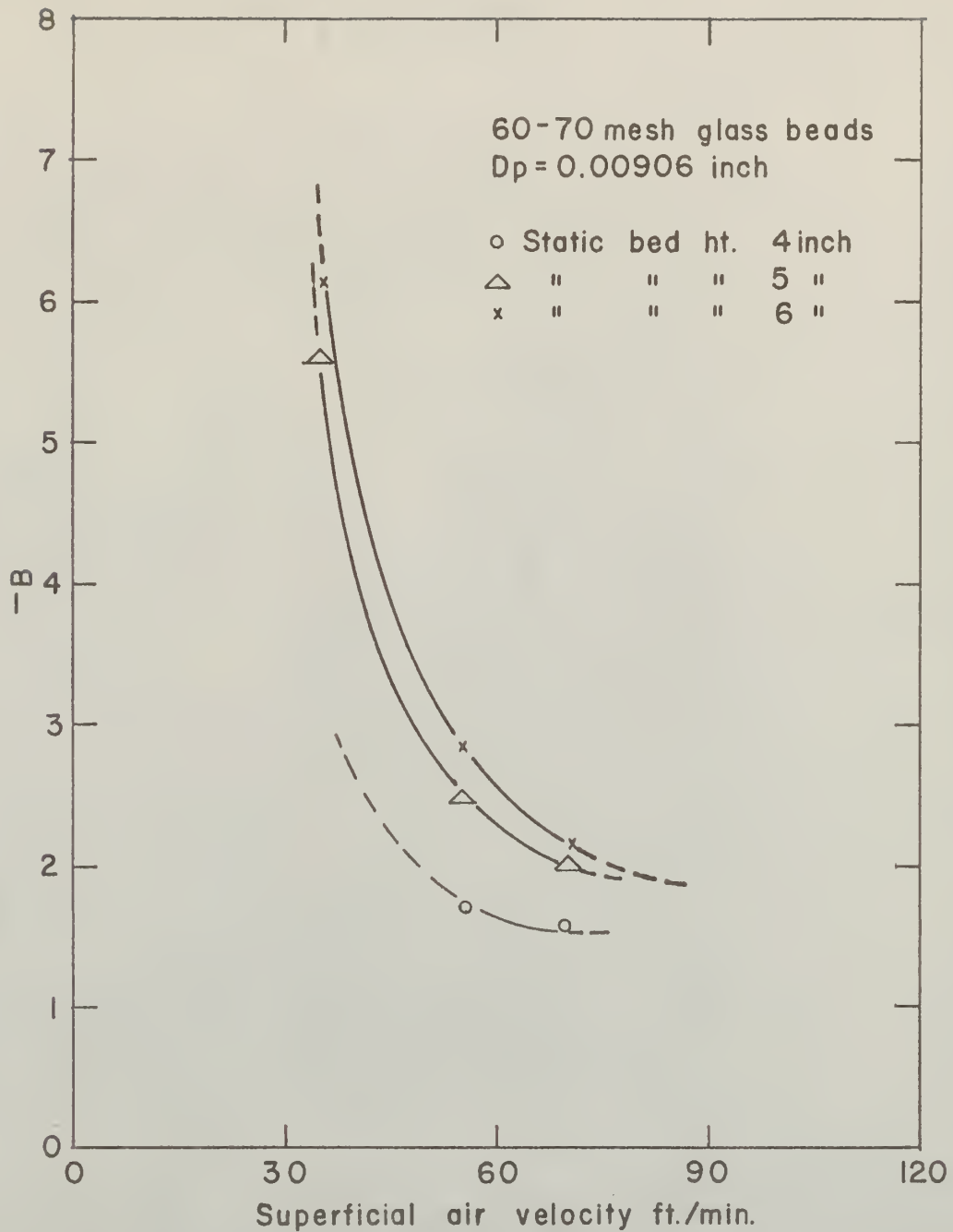


Fig. 17-(b) Correlation of Index -B v.s. superficial air velocity with static bed height as parameter.

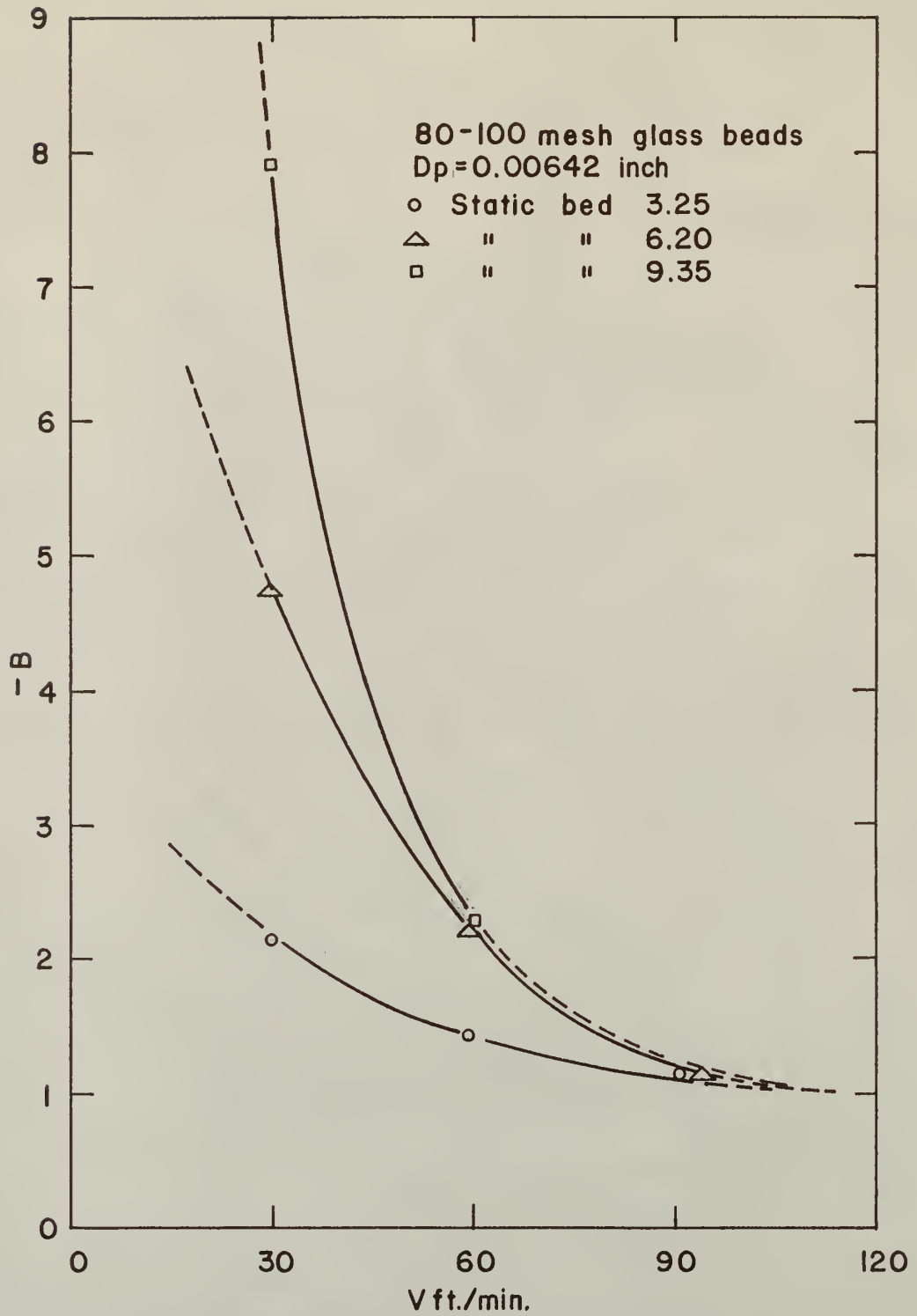


Fig. 17-(c) Correlation of Index-B v.s. air velocity with static bed height as parameter.

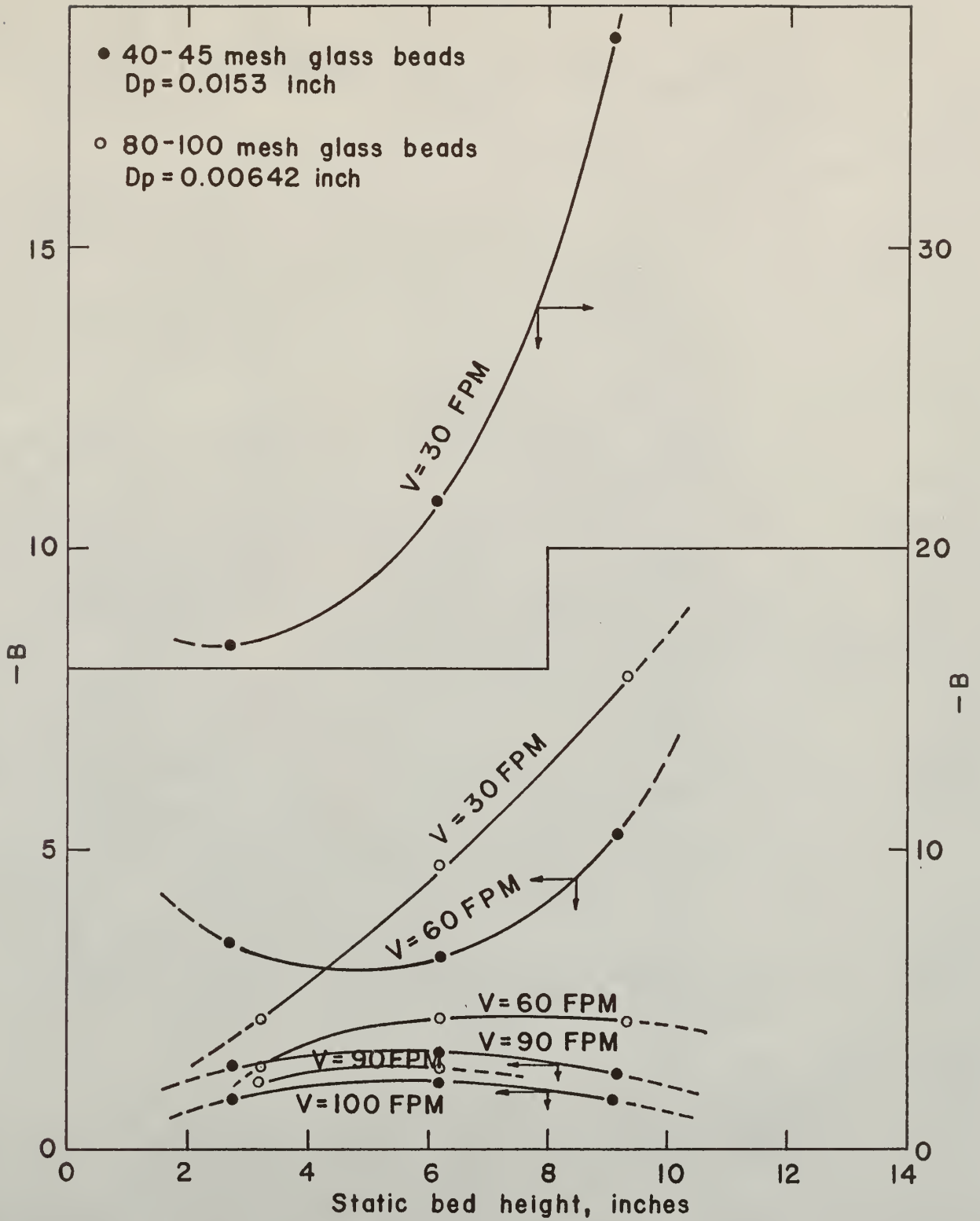


Fig. 18. Correlation of Index -B vs. static bed height with air velocity as parameter.

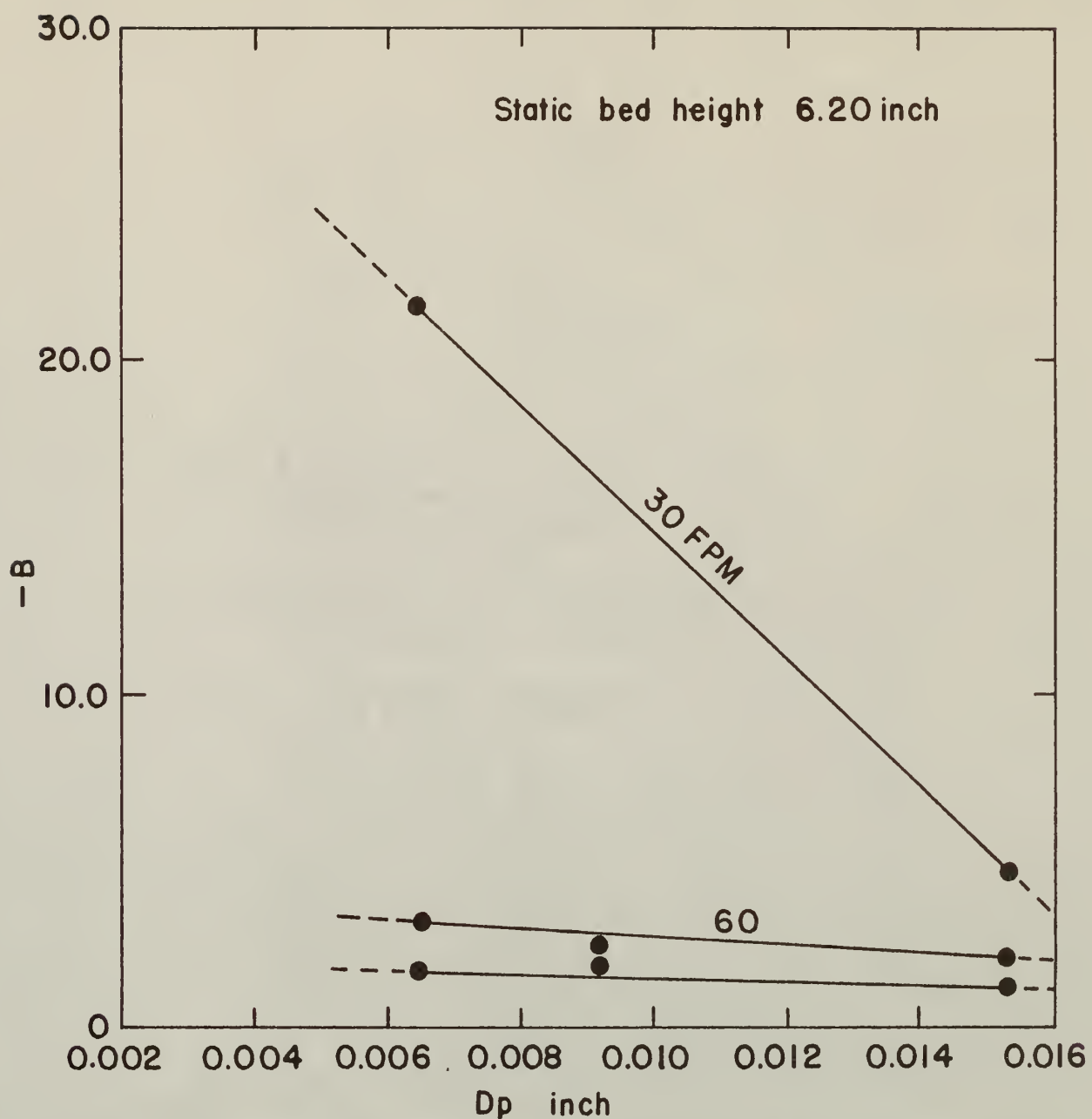


Fig. 19. Correlation of index $-B$ v.s. D_p for same static bed height, 6.20 inch and fluidizing at three different air velocity.

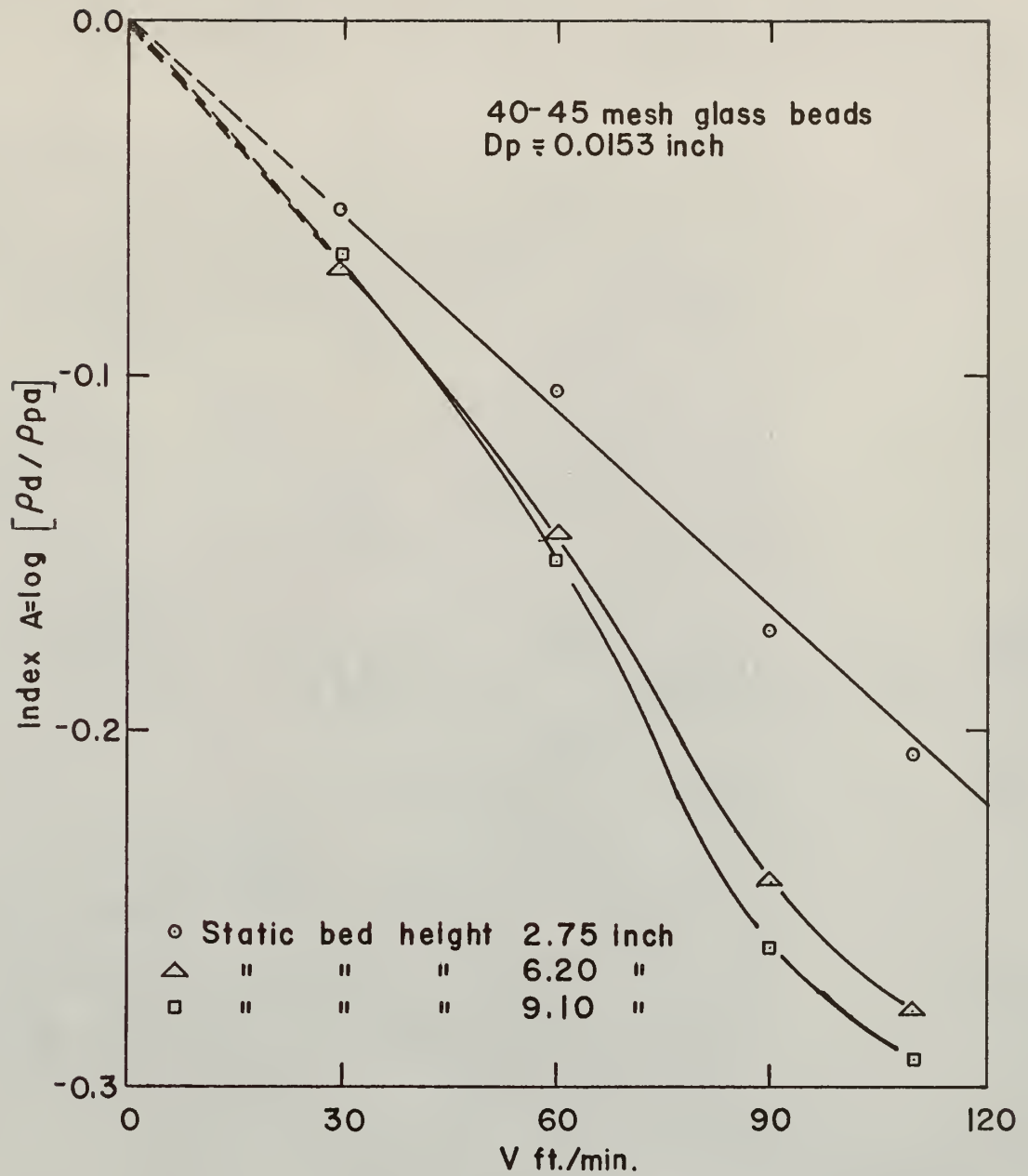


Fig. 20-(a) Correlation of Index A v.s. air velocity with static bed height as parameter.

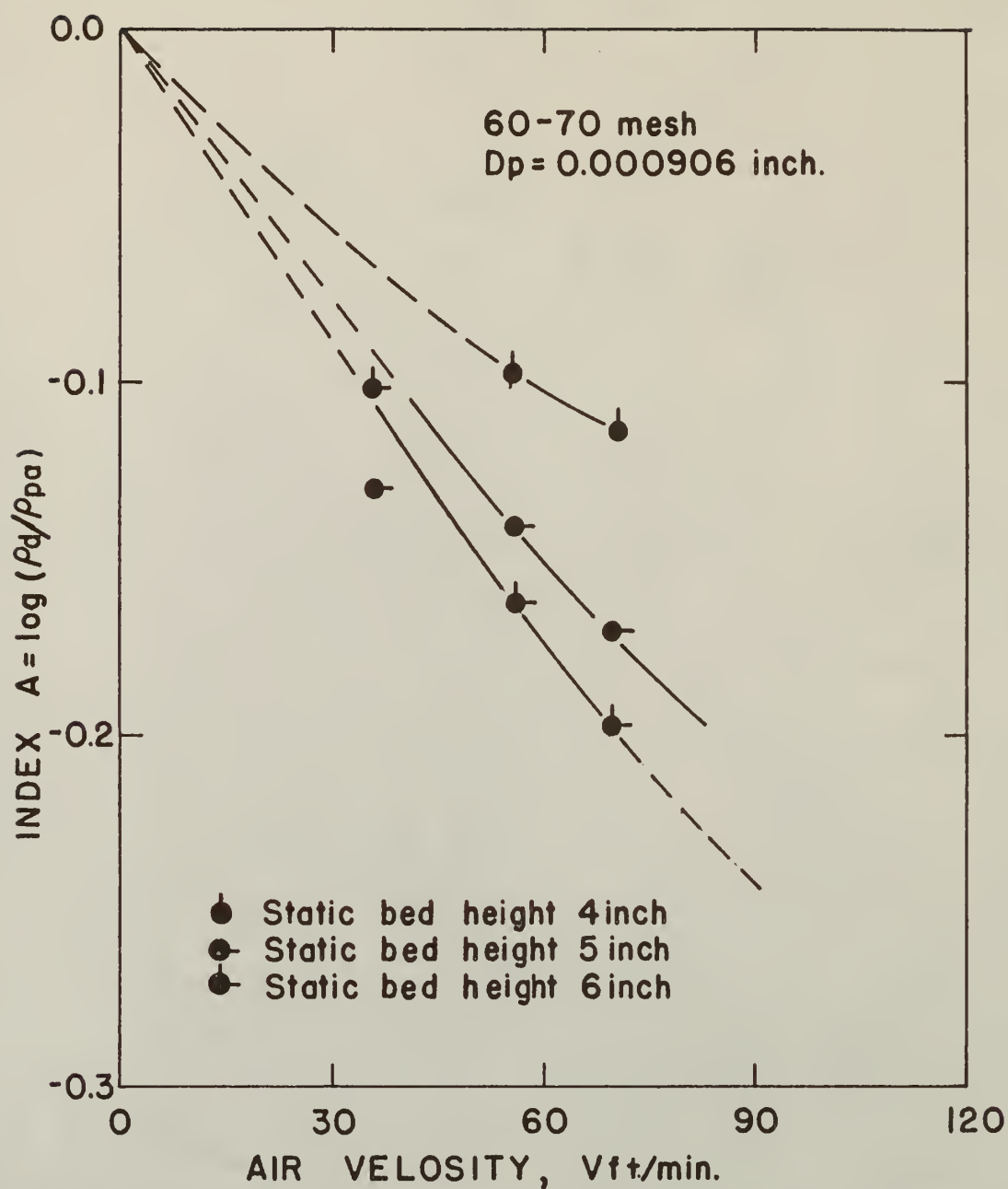


Fig. 20-(b). Index-A v.s. air velocity with static bed as parameter.

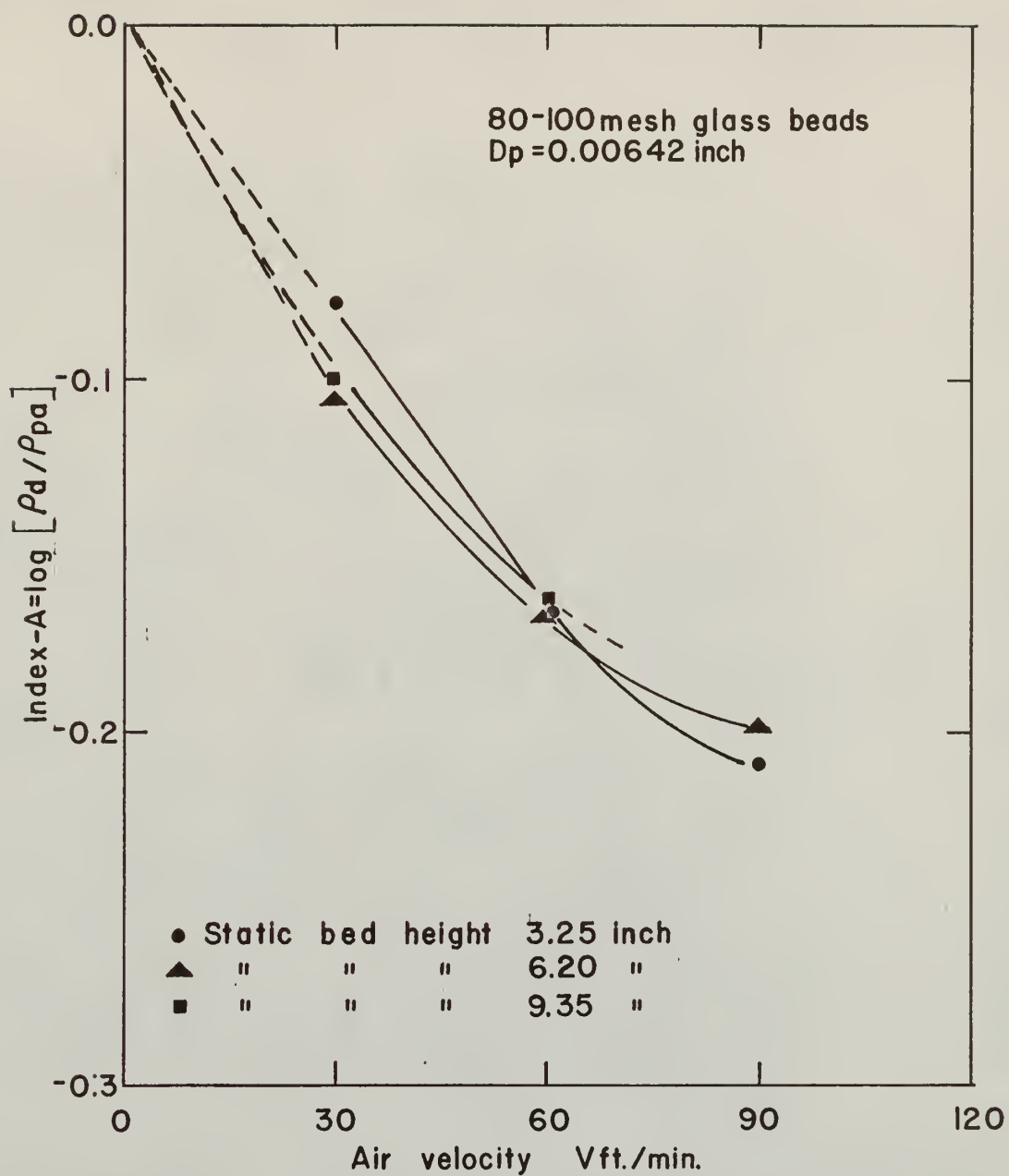


Fig. 20-(c) Correlation of Index -A v.s. air velocity with static bed height as parameter.

They were fluidized at an air velocity of 60 ft/min. The axial density profiles were obtained and index A and index B calculated for every bed composition. Bed compositions of 0, 2, 5, 10, 50, 90, 95, 98 and 100 percent by weight of 40-45 mesh glass beads were used. The results show that the mixed particle size affects index A and index B significantly only when the composition is below 10% or above 90% by weight of 40-45 mesh glass beads.

As the state of aggregative fluidized beds is affected by the three major variables, i.e., superficial fluid velocity, particle size and the ratio of static bed height to the diameter of column; index B as well as index A (which is used to describe the state of aggregative fluidization) should be correlated in a single empirical equation. By applying the Pi-theorem in the dimensional analysis, we obtain:

$$|A| = \alpha' \left(\frac{V D_p \rho_F}{\mu} \right)^a (D_p/D_c)^b (L_{mf}/D_c)^c \quad (20)$$

$$|B| = \beta' \left(\frac{V D_p \rho_F}{\mu} \right)^{a'} (D_p/D_c)^{b'} (L_{mf}/D_c)^{c'} \quad (21)$$

where, the values of α' , a , b , c ; β' , a' , b' and c' can be obtained from the experimental data.

The equations for the series of data obtained are:

$$|A| = 1.396 \times 10^{-5} \left(\frac{D_p V \rho_F}{\mu} \right)^{0.978} (D_p/D_c)^{-1.265} (L_{mf}/D_c)^{0.3469} \quad (22)$$

$$|B| = 5.492 \times 10^9 \left(\frac{D_p V \rho_F}{\mu} \right)^{-2.094} (D_p/D_c)^{3.045} (L_{mf}/D_c)^{0.3474} \quad (23)$$

Equations (22) and (23) are plotted in Fig. 23 and Fig. 24 in order to compare the calculated values for $|B|$ and $|A|$ with those experimentally determined (Table 4 and Table 5).

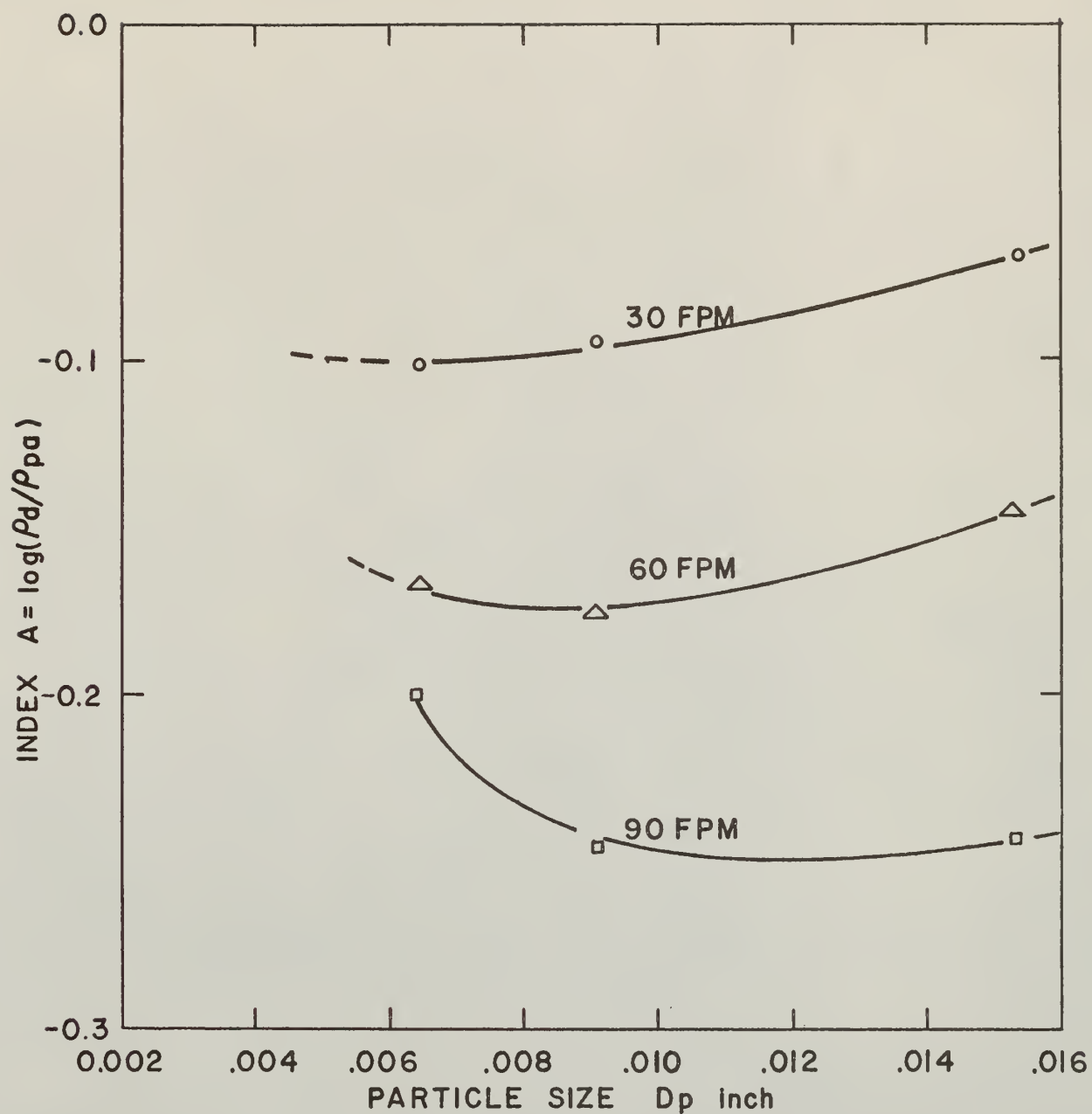


Fig. 21. Index-A v.s. D_p for static bed height = 6.20 inch fluidizing at various velocity.

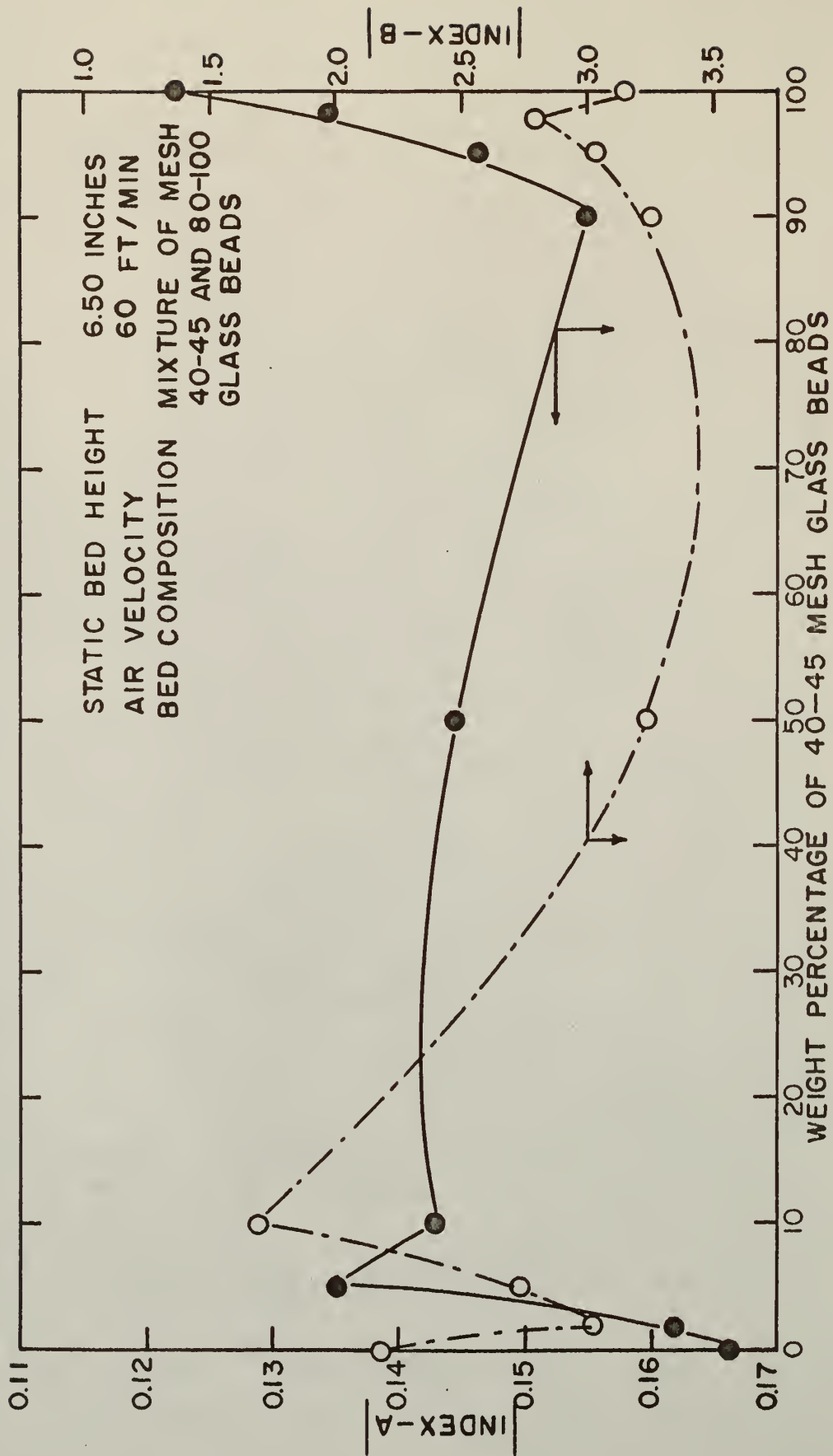


Fig. 22. Effect of bed composition on indices A and B.

Table 4. Comparison of calculated | B | with the experimental data.

40-45 mesh glass beads		:	80-100 mesh glass beads	
Experimental	Calculated	:	Experimental	Calculated
16.789	12.524		25.695	18.910
3.438	2.932		2.925	4.382
1.391	1.264		1.572	1.391
0.813	0.830		1.710	2.231
21.471	16.009		5.602	6.298
3.151	3.778		2.450	2.496
1.645	1.635		2.035	1.560
1.133	1.074		6.192	6.660
36.984	18.548		60-70 mesh glass beads	
5.204	4.358		2.850	2.637
1.213	1.864		2.142	1.651
0.815	1.253		2.141	5.929
10.395	13.280		1.406	1.388
3.328	3.102		1.135	0.594
1.033	1.264		4.766	7.642
24.010	17.444		2.185	1.762
3.151	4.051		1.238	0.762
1.150	1.710			

Table 5. Comparison of calculated | A | with the experimental data.

40-45 mesh glass beads		:	80-100 mesh glass beads	
Experimental	Calculated	:	Experimental	Calculated
0.0535	0.0523		0.0793	0.0704
0.1052	0.1030		0.1667	0.1386
0.1722	0.1526		0.2074	0.2061
0.2072	0.1857		0.1055	0.0868
0.0691	0.0705		0.1663	0.1723
0.1452	0.1384		0.1993	0.2548
0.2423	0.2047		0.0983	0.1010
0.2789	0.2491		0.1624	0.1990
0.0677	0.0800		60-70 mesh glass beads	
0.1525	0.1574		0.1142	0.1625
0.2617	0.2341		0.0991	0.1304
0.2912	0.2818		0.1318	0.0899
0.0463	0.0554		0.1420	0.1386
0.0825	0.1092		0.1716	0.1726
0.1575	0.1661		0.1029	0.0961
0.0689	0.0702		0.1629	0.1482
0.1223	0.1388		0.1987	0.1844
0.2151	0.2072			
0.0802	0.0806			
0.1744	0.1596			

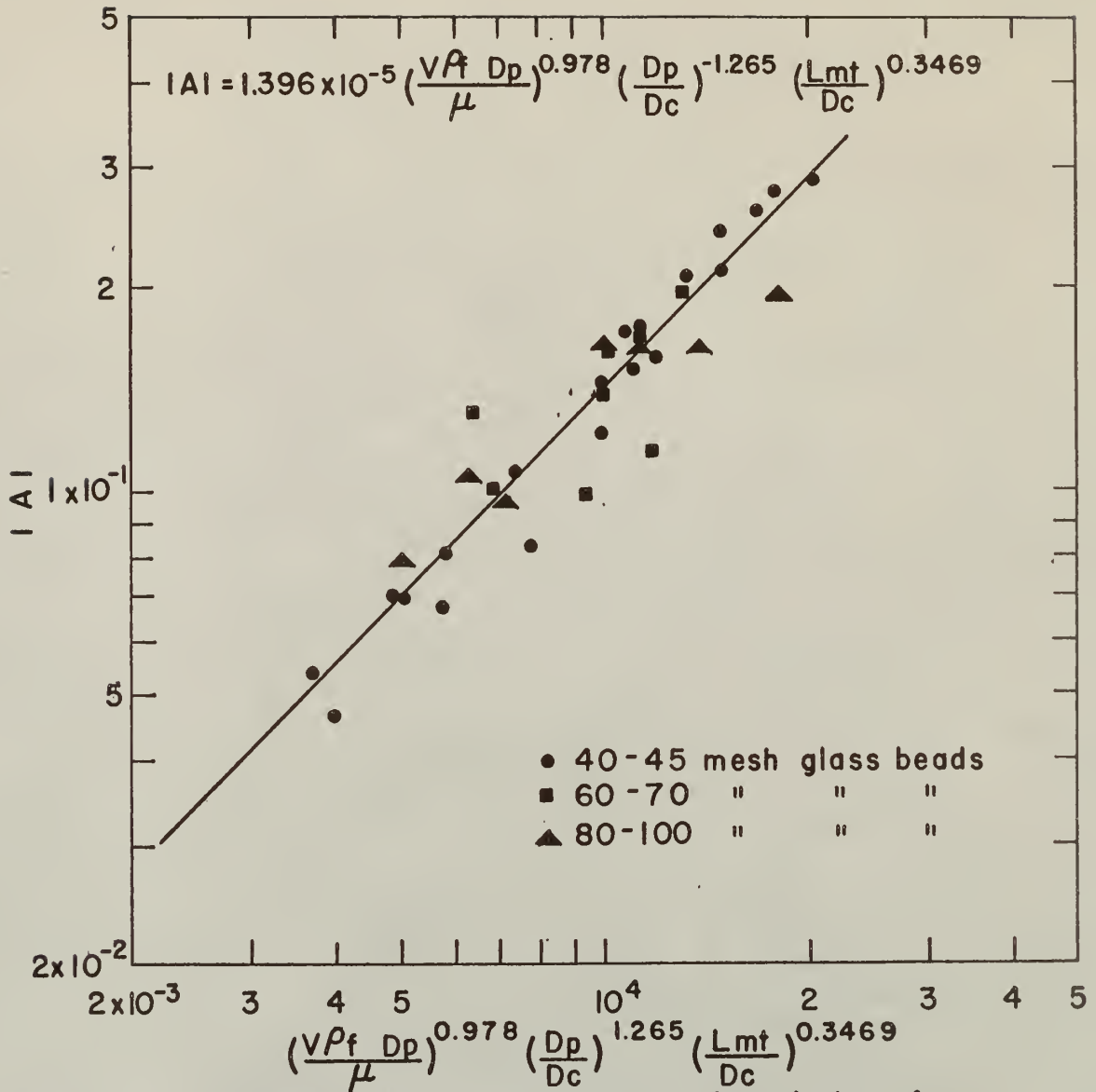


Fig. 23. Dimensionless correlation for Index A.

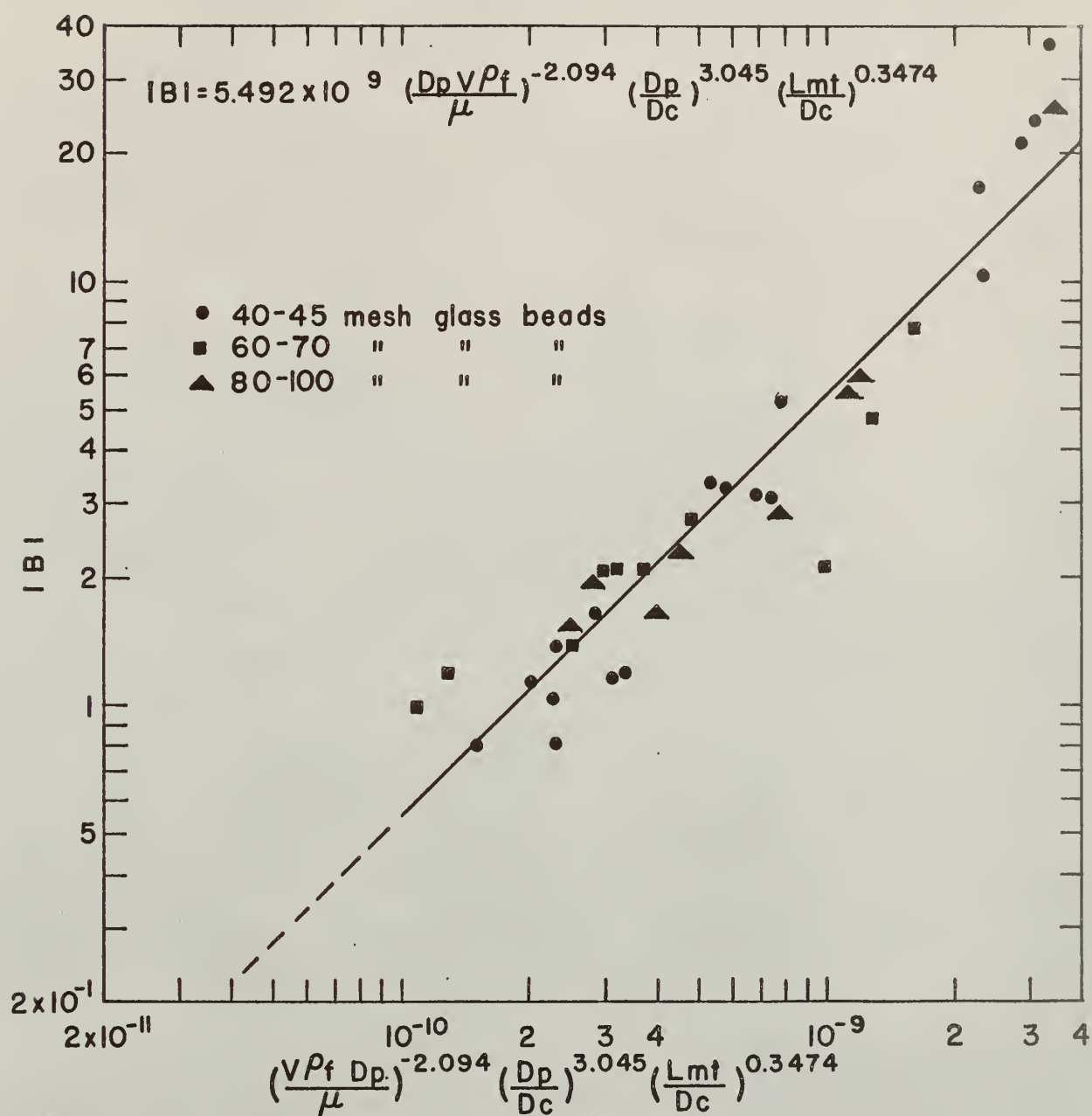


Fig. 24. Dimensionless correlation for Index B.

We can now calculate a complete axial density profile for the air-particle fluidizing system with Fig. 15 and equations for $|A|$ and $|B|$. It is illustrated in the following illustration:

Illustration:

In a fluidization test, glass beads of 40-45 mesh were used (Run. P40-06), experiment was run at room temperature and with air at 1 atmospheric pressure in a circular column of 4 inch diameter. The bed had a height of 6.20 inch at incipient fluidization. Estimate the fluidized bulk density profile, for the bed fluidizing at 1 ft/sec. of superficial air velocity. Compare it with the experimental data.

Solution:

Properties of air at the fluidizing condition:

$$\begin{aligned}\rho_F &= 0.0757 \text{ lb/cu-ft} \\ \mu &= 1.23 \times 10^{-5} \text{ lb/ft-sec.}\end{aligned}\tag{11}$$

The air velocity at incipient fluidization can be calculated:

$$V_{mf} = 22.35 \text{ ft/min.}$$

Properties of particles:

$$\begin{aligned}D_p &= 0.0153 \text{ inch} \\ \rho_{pa} &= 1.502 \text{ gm/cu-cm}\end{aligned}$$

Constant density zone:

$$V = 1 \text{ ft/sec.} = 60 \text{ ft/min.}$$

$$V - V_{mf} = 37.65 \text{ ft/min.}$$

$$\text{from Fig. 15 (a), } \bar{\rho}_d = 1.168 \text{ gm/cu-cm.}$$

Falling density zone:

$$N_{Re} = \frac{V \rho_F D_p}{\mu} = \frac{(1)(0.0757)(0.0153)}{(1.23 \times 10^{-5})(12)} = 7.85$$

$$(N_{Re})^{0.978} \left(\frac{D_p}{D_c}\right)^{-1.265} \left(\frac{L_{mf}}{D_c}\right)^{0.3469} = (7.85)^{0.978} \left(\frac{0.0153}{4}\right)^{-1.265} \left(\frac{6.20}{4}\right)^{0.3469}$$

$$= (7.49)(1.120)(1.164) = 9.763 \times 10^3$$

From Fig. 23,

$$|A| = 0.139$$

$$A = \log (P_d / \rho_{pa}) = -0.139$$

$$(N_{Re})^{-2.094} \left(\frac{D_p}{D_c}\right)^{3.045} \left(\frac{L_{mf}}{D_c}\right)^{0.3474} = (7.85)^{-2.094} \left(\frac{0.0153}{4}\right)^{3.045} \left(\frac{6.20}{4}\right)^{0.3474}$$

$$= (1.34 \times 10^{-2})(4.335 \times 10^{-8})(1.167)$$

$$= 6.765 \times 10^{-10}$$

From Fig. 24,

$$|B| = 3.65$$

$$B = -3.65$$

Thus, the density in the falling density zone can be calculated by the following equation:

$$\log (\rho / \rho_{pa}) = A + B (h/L_{mf} - 1)^2$$

i.e.,

$$\log (\rho / 1.502) = -0.139 - 3.65 (h/6.2 - 1)^2$$

The calculated density and the experimental values for the falling density zone are listed as follows:

Height from distributor h, in	Experimental		Calculated	
	ρ / ρ_{pa}	gm/cu-cm	ρ / ρ_{pa}	gm/cu-cm
7.00	0.614	0.922	0.633	0.951
8.00	0.344	0.516	0.358	0.538
9.00	0.153	0.229	0.132	0.200
10.00	0.039	0.058	0.031	0.047
11.00	0.020	0.030	0.005	0.007

The experimental data are plotted in Fig. 25 with the density profile calculated by the obtained equation.

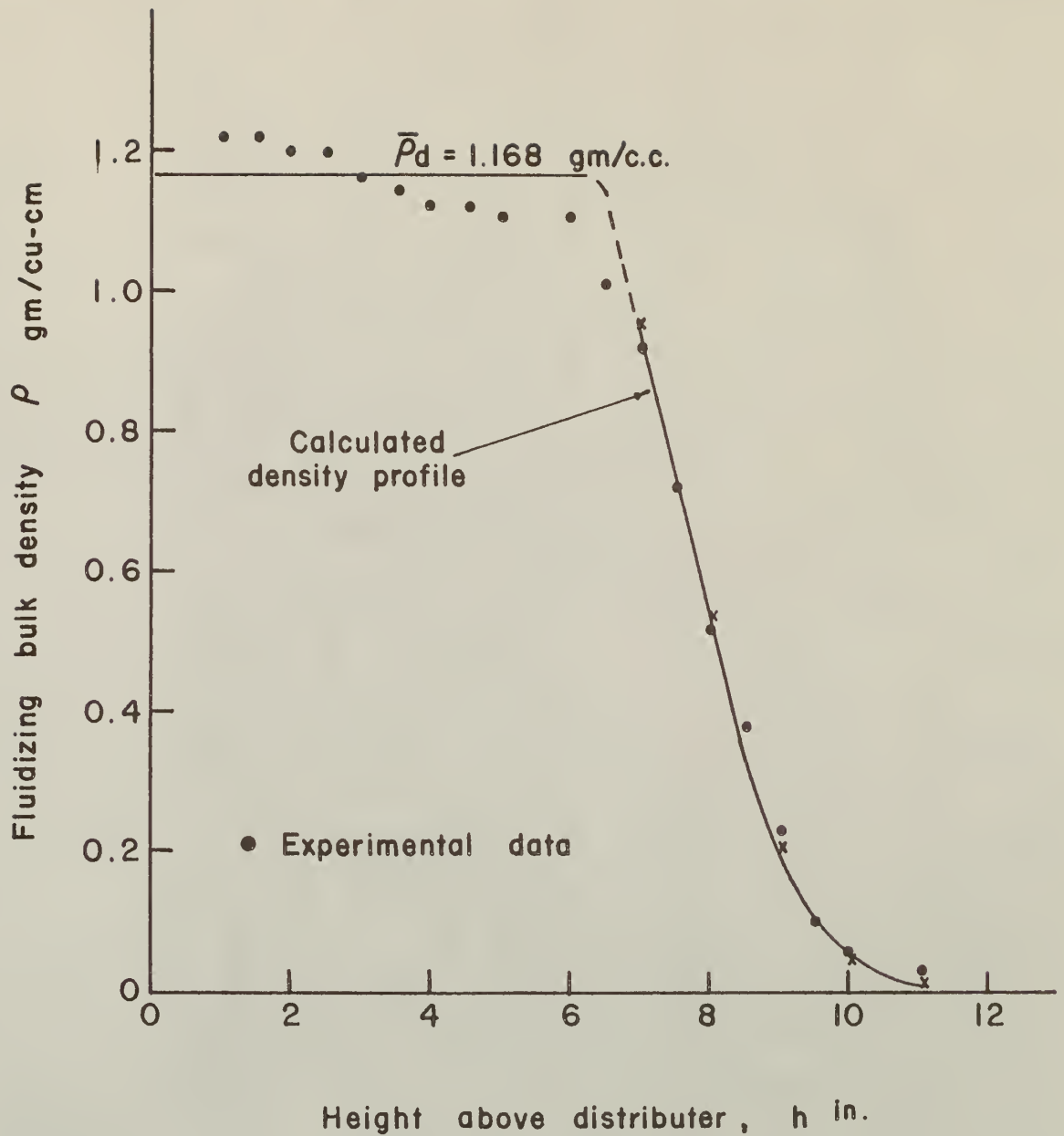


Fig. 25. Comparison of experimental data with calculated density profile for the illustration

Determination of the Fluidized Bed Height

The behavior of gas-solid fluidized beds observed in this work deviated considerably from the behavior of an ideal expansion of a packed bed. The particles were not as uniformly suspended in the bed as is usually observed in most of the liquid fluidized beds. Even the two phase model for gas-solid fluidization is only a very approximate one. Actually, there is no distinct phase which can be defined as a bubble phase consisting of purely gases, nor is there a dense phase consisting of uniform emulsion of particles. The bubbles, formed by the gas and held in the pore spaces between solids, grow in size as they rise from gas distributor toward the top of the bed, and finally burst near the position corresponding to the static bed height. The small size particles suspended in the air stream were fluctuating and weaving at the top of the bed, and so the disperse phenomenon dominated the top layer of the bed. Thus, there is no clear height of the fluidized bed that can be measured visually.

Three methods can be used to measure the fluidized bed height from the solid density profile under measurement.

(a). End point of the density profile:

Tracing the density profile in the falling density zone and extrapolating to the position at which the density diminishes to zero, we can define the fluidized bed height at this point. This height should correspond to the maximum height of the fluidized bed as observed visually by previous investigators.

(b). Cross point of the normalized density profile:

As indicated in the dimensionless density profiles, Figure 13, the significant height of aggregative fluidized beds can be defined at the point where

the ideally expanded density profile crosses the actually developed density profile of the fluidized bed. The particles are assumed to be uniformly distributed below this height, and the portion above it is less significant to the over-all performance of the total bed and is, therefore, considered to be negligible. This height, as defined, should signify the bed, though aggregative, as a particulate one.

(c). Height determined by IIS methods:

The fluidized bed height - defined as the breaking point of the axial IIS distribution curve - was proposed by Steward (25). However this height corresponds closely to the height determined by method (b).

It is expected that the bed defined by method (b) or method (c) would give better correlation than by method (a), if the analogous correlation of a particulate fluidized bed is applied in an aggregative fluidized bed. In the calculation for reactor conversion, the bed height defined in (b) and (c) should give a more realistic picture, since the insignificant part of the disperse phase is neglected in this consideration.

The fractional fluidized bed expansion $(L - L_{mf})/L_{mf}$, calculated from the height as determined by method (a) was plotted as a function of superficial air velocity with L_{mf}/D_c as parameters in Fig. 26. This correlation shows a linear relationship, which is consistent with the data presented by Minet et.al. for chars (17). The effect of the static bed height on the expansion of fluidized beds is also shown in this figure. Similar correlation was attempted with the bed height defined by method (b) or method (c). The result shows that it deviates considerably from the linear relationship (Fig. 27). Fig. 28 is a comparison between the fluidized bed height determined by method (a) and method (c). It indicates that for a certain value of L_{mf}/D_c and particle

size, these two defined heights are different from each other by a proportionality factor.

Pressure Gradient Correlations

The pressure-drop-flow diagram in a fluidizing system is well explained in the following paragraph (6):

If the behavior of packed bed is followed when a gas stream is introduced from the bottom, the pressure drop at first increases with flow rate without bed expansion. When the flow rate is sufficient to cause a pressure gradient equal to the buoyant weight of the solids per unit volume, further increase in the flow rate causes the bed to expand -- i.e. increase the fractional void volume -- the pressure drop remains substantially constant....

The two pressure-drop-flow diagrams obtained are shown in Figs. 28-a. and 28-b. They resemble in shape to the pressure-drop-flow diagram for ideally fluidizing systems (11), i.e., the pressure drop becomes essentially just the weight of the bed at the air velocity just beyond the minimum velocity of fluidization. This probably indicates that the fluidizing bed developed with the air distributor (canvas filter cloth) was macroscopically almost equivalent to the ideal expansion of a packed bed, since the size of air bubbles formed was small and the bubbles were nicely distributed in the bed at the air velocity around the minimum velocity of fluidization. This also implies that the tube-bundle theory (4) developed for the study of the pressure drop through a packed bed may be applied analogously to the analysis of the pressure gradient in the present fluidizing systems, provided the fluidized bed height can be correctly determined.

As mentioned previously, when the air velocity reaches the minimum velocity of fluidization, the pressure drop, ΔP is constant and substantially equal

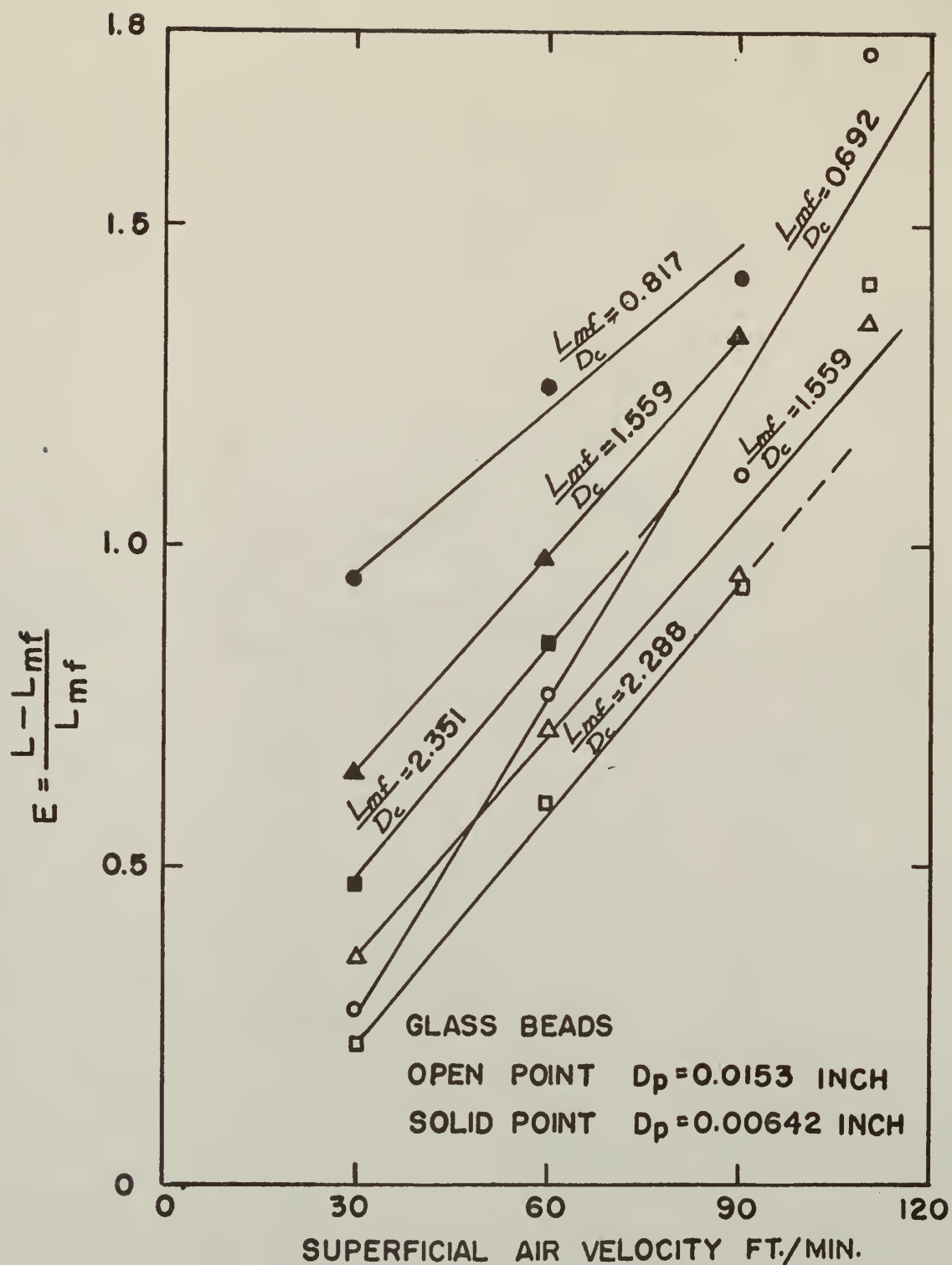


Fig. 26. The fractional fluidized bed expansion from height determined by method (a).

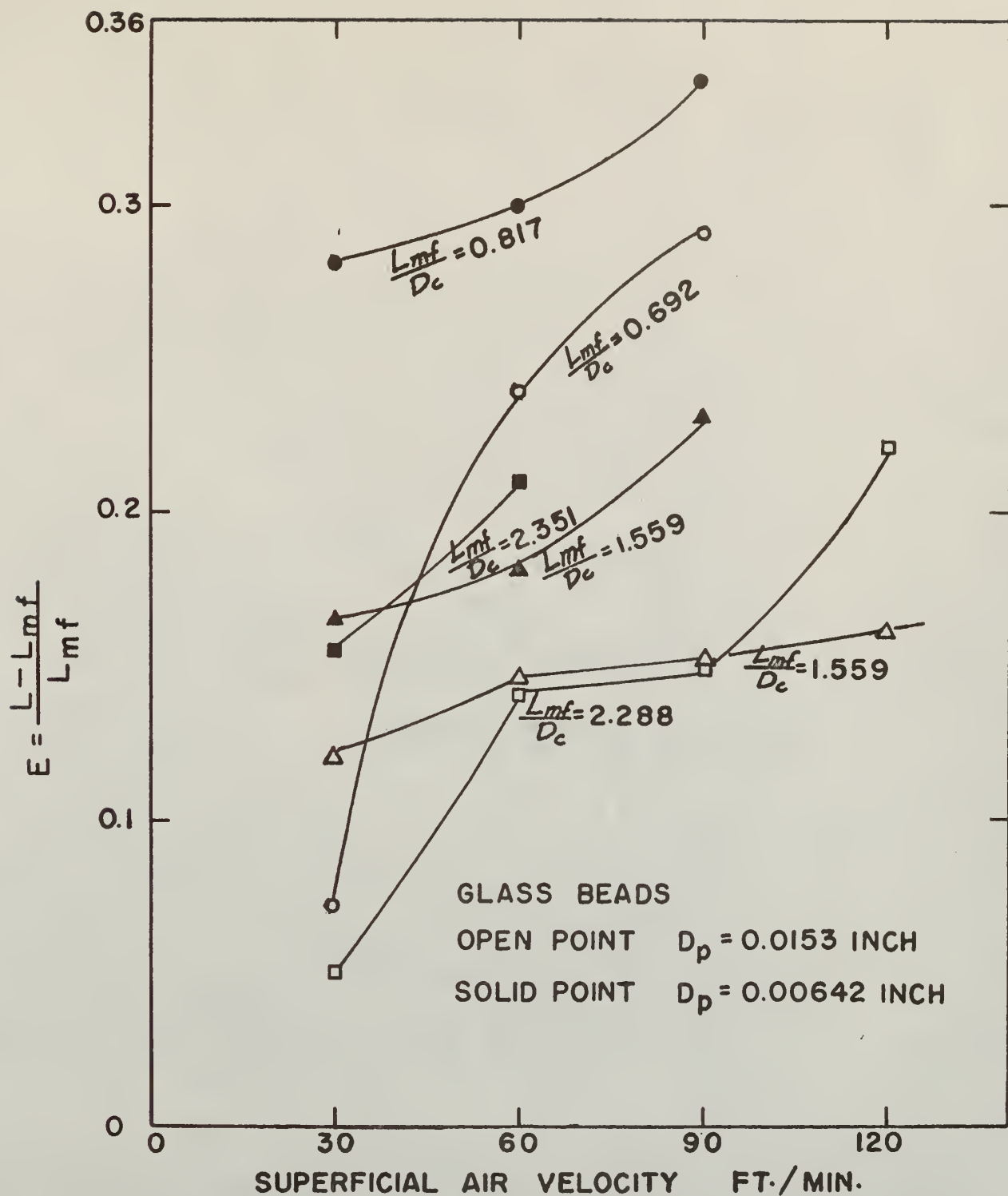
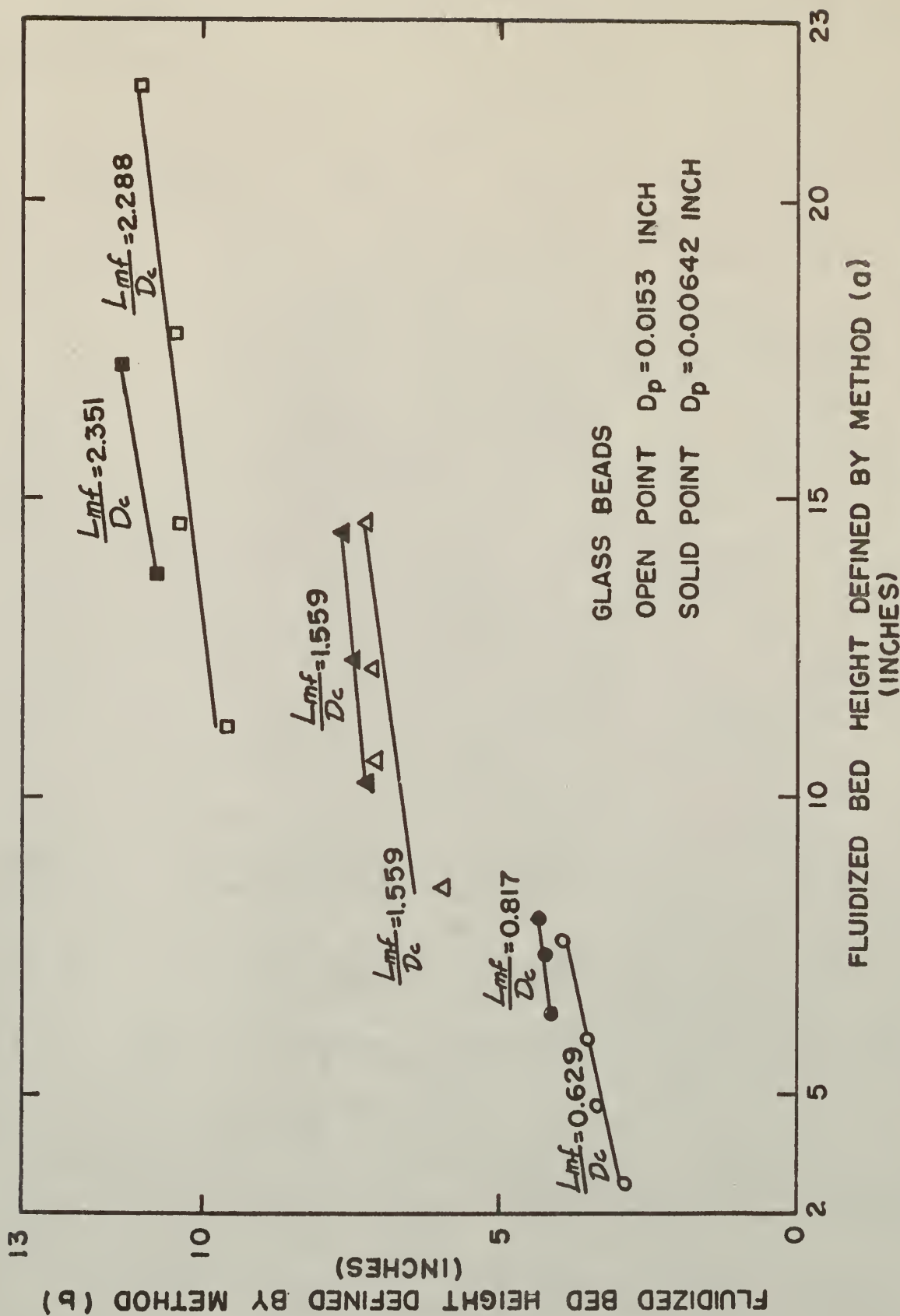


Fig. 27. The fractional fluidized bed expansion from height determined by method (b)



to the weight of the particles constituting the bed. But the pressure gradient is varied with increased air velocity, since the air flow beyond the minimum fluidizing velocity causes the bed to expand. Thus, the pressure gradient $\Delta P/L$ decreases with the increase in the air velocity.

In Fig. 29, the pressure gradients, $\Delta P/L$, are correlated in a form of friction factor, f_k , originally proposed (6) for correlating the pressure drops through packed beds, against the modified Reynolds number, $N_{Re}' = \frac{D_p V \rho_F}{\mu} \cdot \frac{1}{1 - \epsilon}$. But, the numerical values of the pressure gradient, $\Delta P/L$, for fluidized beds should depend on the definition of fluidizing bed heights used. (See previous section.) The correlation obtained with the height determined by method (a) yields the curves resembling the Ergun equation for packed bed in shape (4), and have varying slopes. Whereas, the correlation with that determined by method (b), like Blake-Kozeny equation, has a constant slope. In both cases the observation of Fig. 29 and the data shows that the effect of particle size, D_p , appears to be more significant than that suggested by the correlations proposed for packed beds. The experimental data indicate that the static bed height also has some effect on the correlation of pressure gradients. But, this factor does not come into the consideration for packed beds.

The results of the dimensionless analysis lead to the following form (Appendix II):

$$f_k = \frac{g_c \Delta P}{L} \cdot \frac{\epsilon^3}{1 - \epsilon} \cdot \frac{D_p}{\rho_F V^2} = \phi \left(\frac{D_p \rho_F V}{\mu} \cdot \frac{1}{1 - \epsilon} \right)^a \left(\frac{D_p}{D_c} \right)^b \left(\frac{L_{mf}}{D_c} \right)^c \quad (24)$$

Where, the constant ϕ , a , b , c , can be evaluated from the experimental data with the height of fluidized bed defined by methods (a) and (b).

With the height defined by method (a), the dimensional analysis gives:

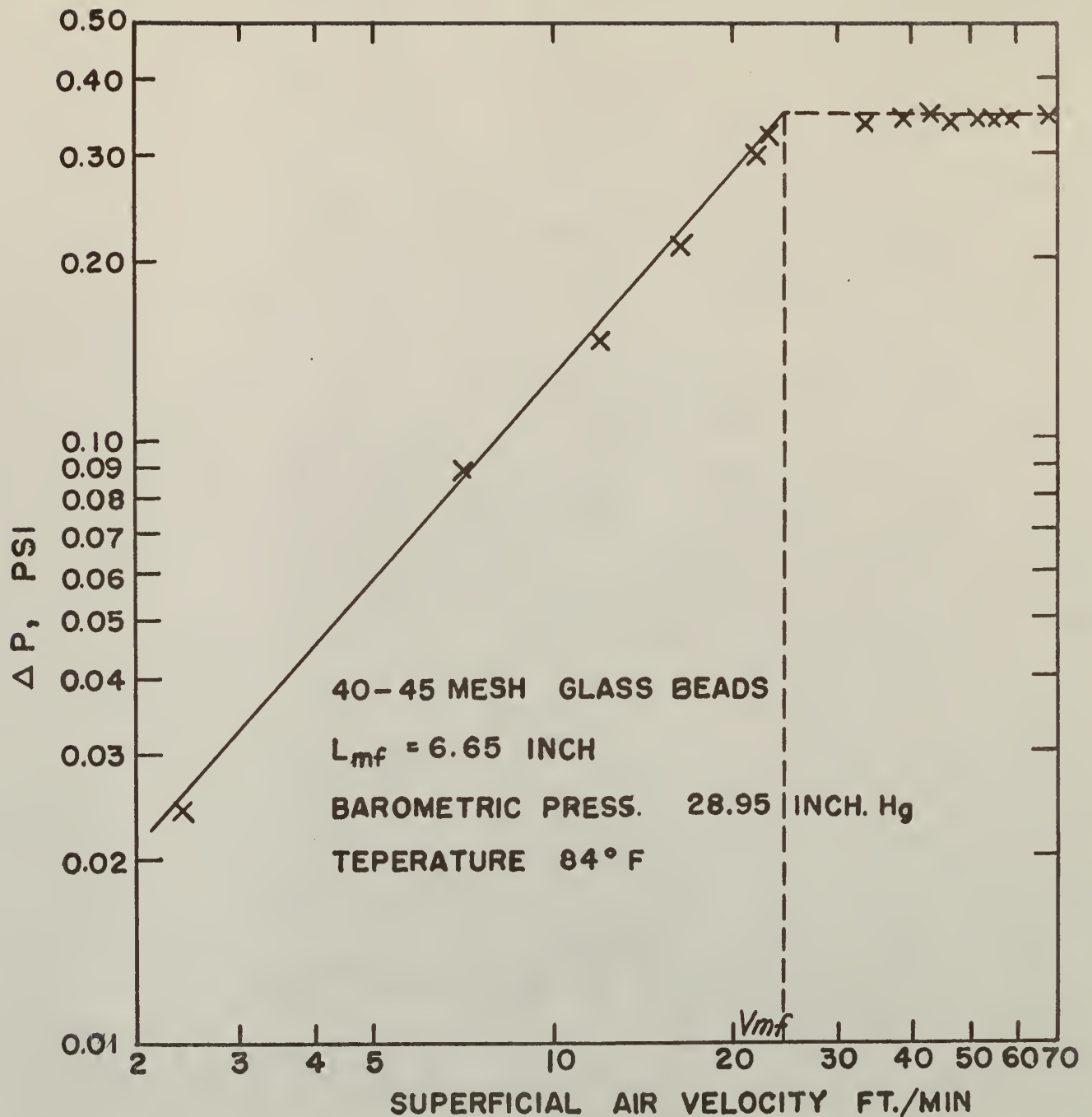


Fig. 28-(a). Pressure-drop-flow diagram for run S40-05.

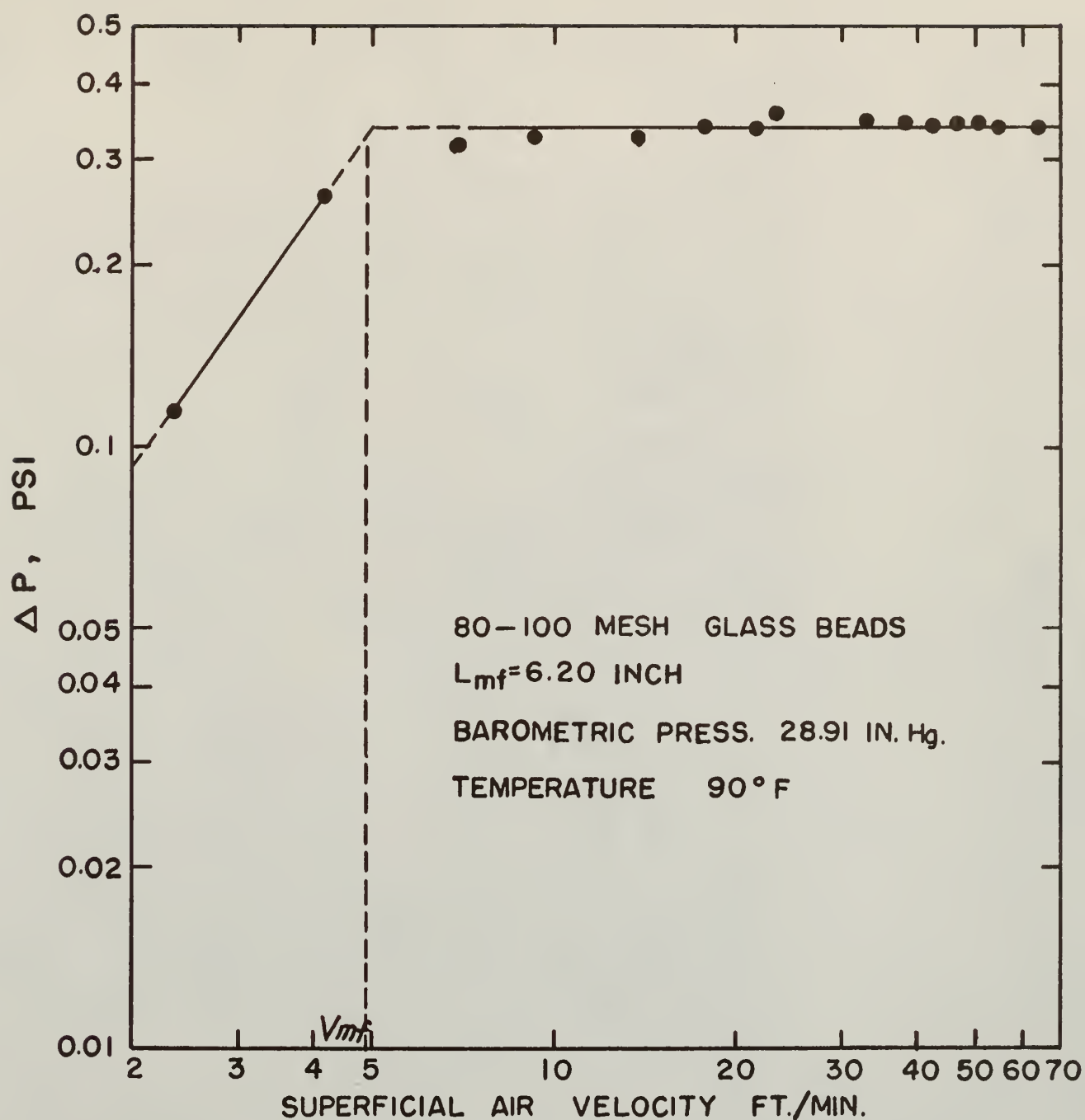


Fig. 28-(b). Pressure-drop-flow diagram for run S80-04.

$$f_K = \frac{g_c \Delta P}{L} \cdot \frac{\epsilon^3}{1 - \epsilon} \cdot \frac{D_p}{\rho_{FV}^2} =$$

$$(9.028 \times 10^5) \left(\frac{D_p \rho_{FV}}{\mu} \cdot \frac{1}{1 - \epsilon} \right)^{-0.945} \left(\frac{D_p}{D_c} \right)^{1.389} \left(\frac{L_{mf}}{D_c} \right)^{-0.224} \quad (25)$$

and, with the height defined by method (b):

$$f_K = \frac{g_c \Delta P}{L} \cdot \frac{\epsilon^3}{1 - \epsilon} \cdot \frac{D_p}{\rho_{FV}^2} =$$

$$(8.80 \times 10^7) \left(\frac{D_p \rho_{FV}}{\mu} \cdot \frac{1}{1 - \epsilon} \right)^{-1.591} \left(\frac{D_p}{D_c} \right)^{2.126} \left(\frac{L_{mf}}{D_c} \right)^{-0.298} \quad (26)$$

Equations (25) and (26) are plotted in Figs. (30) and (31) respectively.

The comparison plot for Equations (25) and 26) is shown in Fig. 32 in order to see which definition of height for fluidized bed gives better results in the proposed correlation for the pressure gradient. It can be seen from this plot that the height determined by method (a) gives more scattered correlation than that determined by method (b). It may be concluded, in the proposed dimensionless correlation of pressure gradient through fluidized bed, that the height determined by method (b) gives more consistent result than that by method (a).

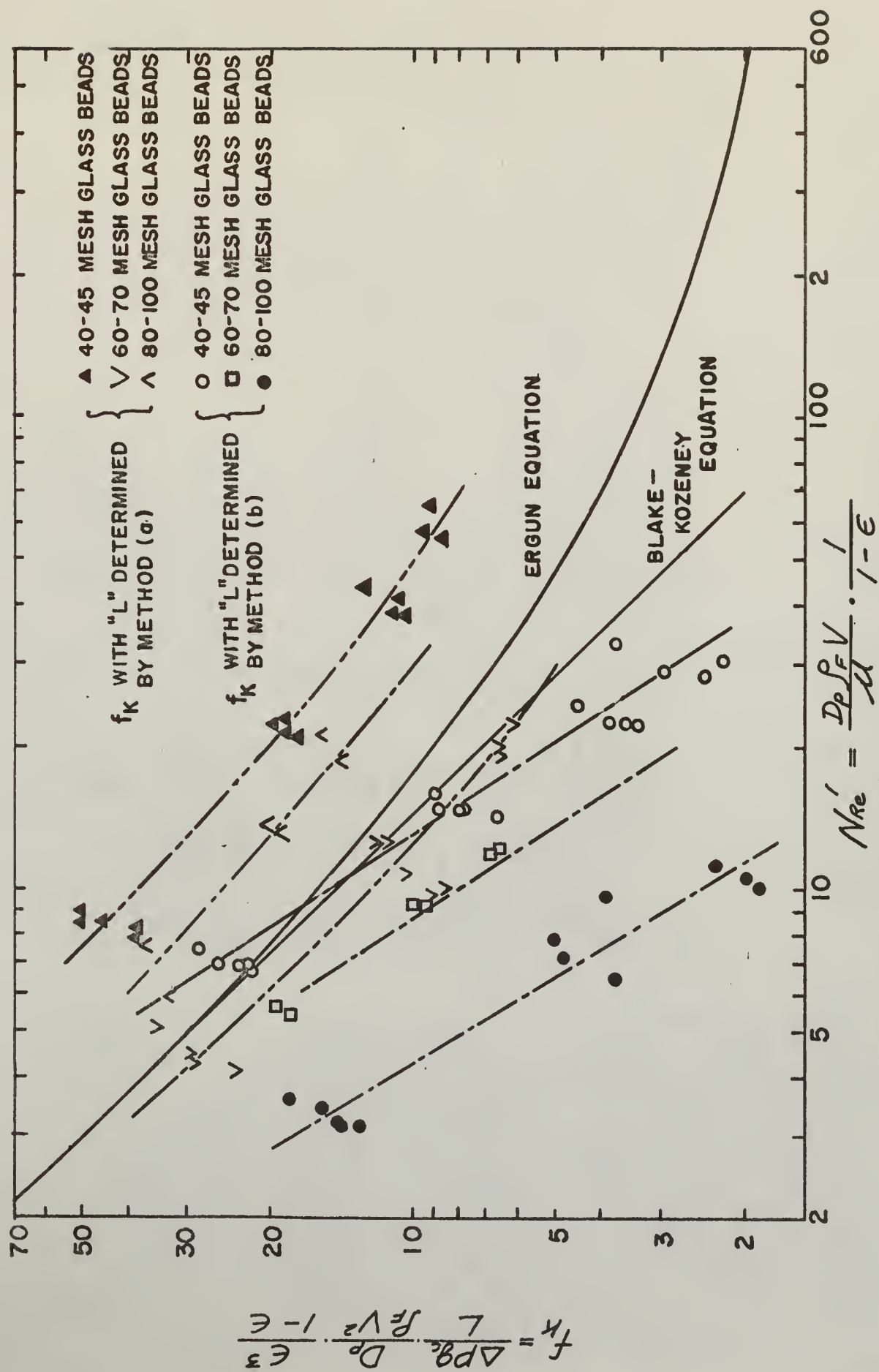


Fig. 29. Friction factor correlation.

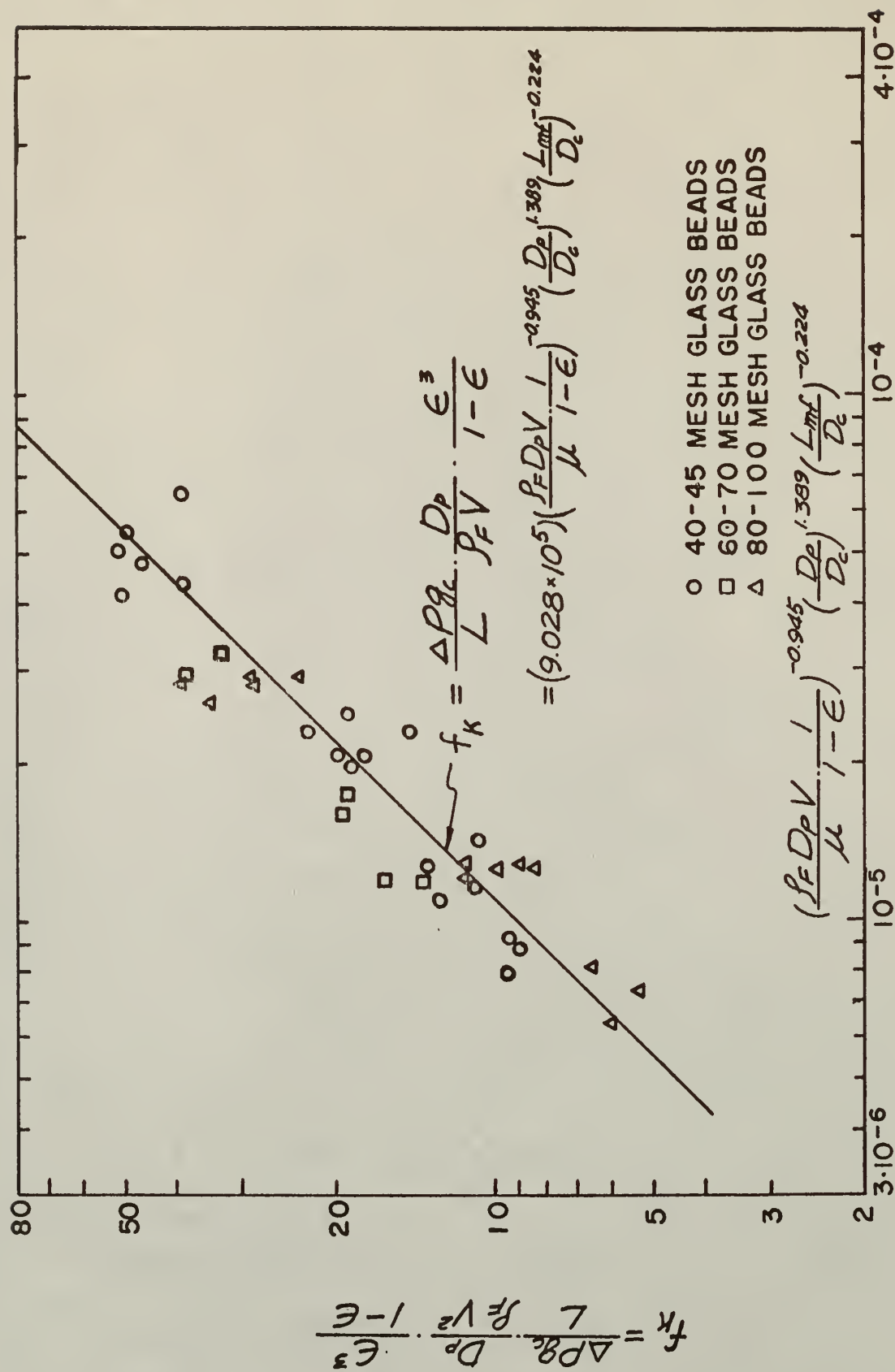


Fig. 30. Dimensionless correlation of friction factor with bed height determined by method (a)

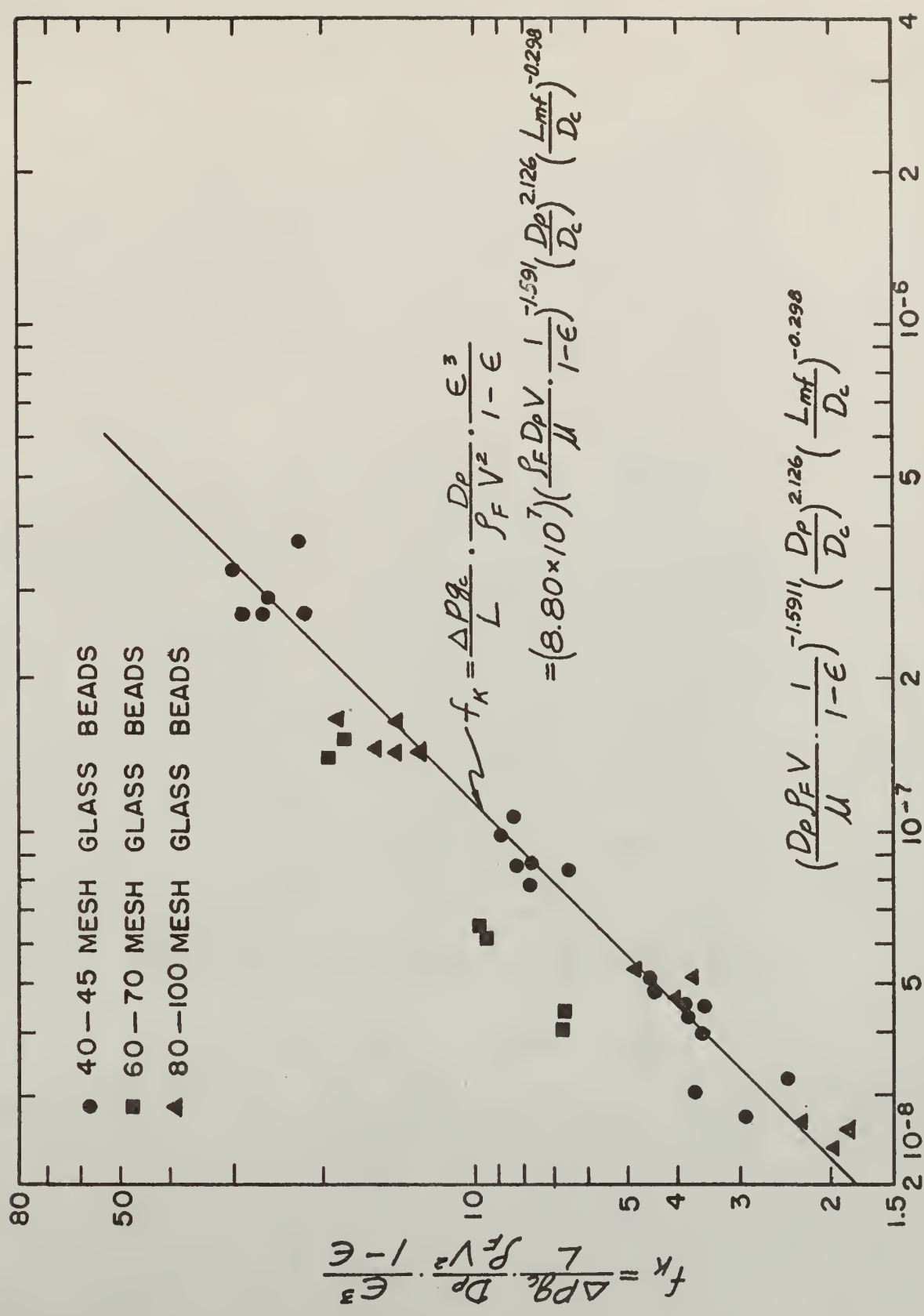


Fig. 31. Dimensionless correlation of friction factor with bed height determined by method (b).

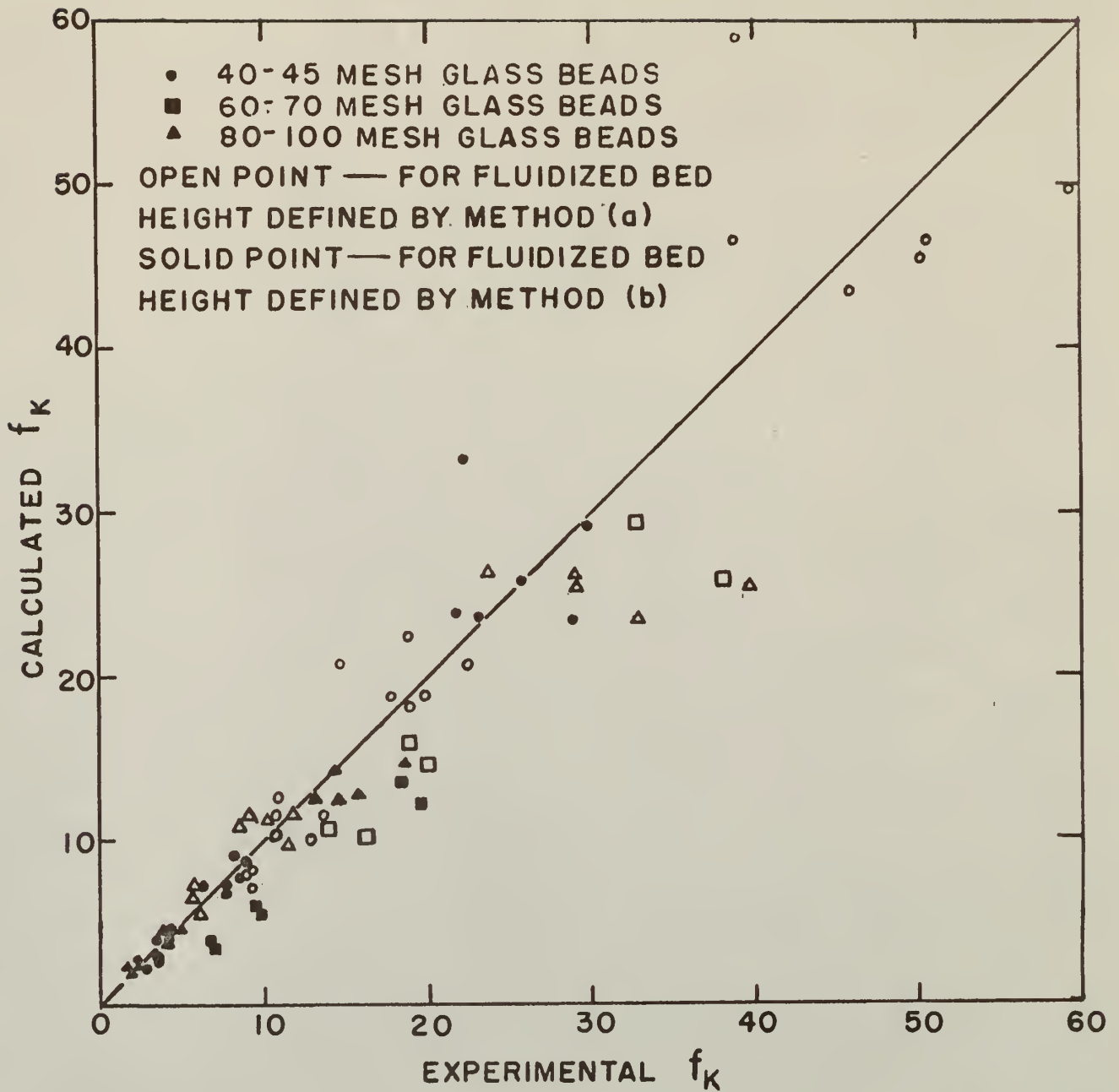


Fig. 32. Comparison of experimental f_K to calculated f_K .

RESULTS AND DISCUSSION

 γ -ray Technique as Applied to the Measurement
of Fluidized Bulk Density in Fluidized Beds

Among the several technical methods developed for the study of local properties of a non-ideal two phase fluid, the γ -ray technique is a relatively new one. It could be applied to the study of the formation and behavior of bubbles in aggregative fluidized beds. By using a short-time constant of the rate-count-meter to obtain the strip chart oscillograms, the size, velocity and frequency of bubbles could be calculated (3). This method also simplified the technical problems in the measurement of local time-average density by using a long-time constant. The capacitance probe method which was used by Dotson (5) and other investigators (18, 1) appeared to disturb the flow pattern of the fluidizing vessel. The capacitance probe inserted in the vessel, though it was small, should change the local flow pattern. The properties investigated were influenced by the probe and should be different from the flow pattern without such disturbance. The γ -ray radiation technique can get rid of this disturbance completely. The radiation passing through the fluidizing vessel has no effect on the motion of fluidizing mixtures.

Comparison of the Short-time Constant
to Long-time Constant Data

Although a 30 second long time constant was used for all the experimental runs, the data obtained were compared with the data taken by short-time

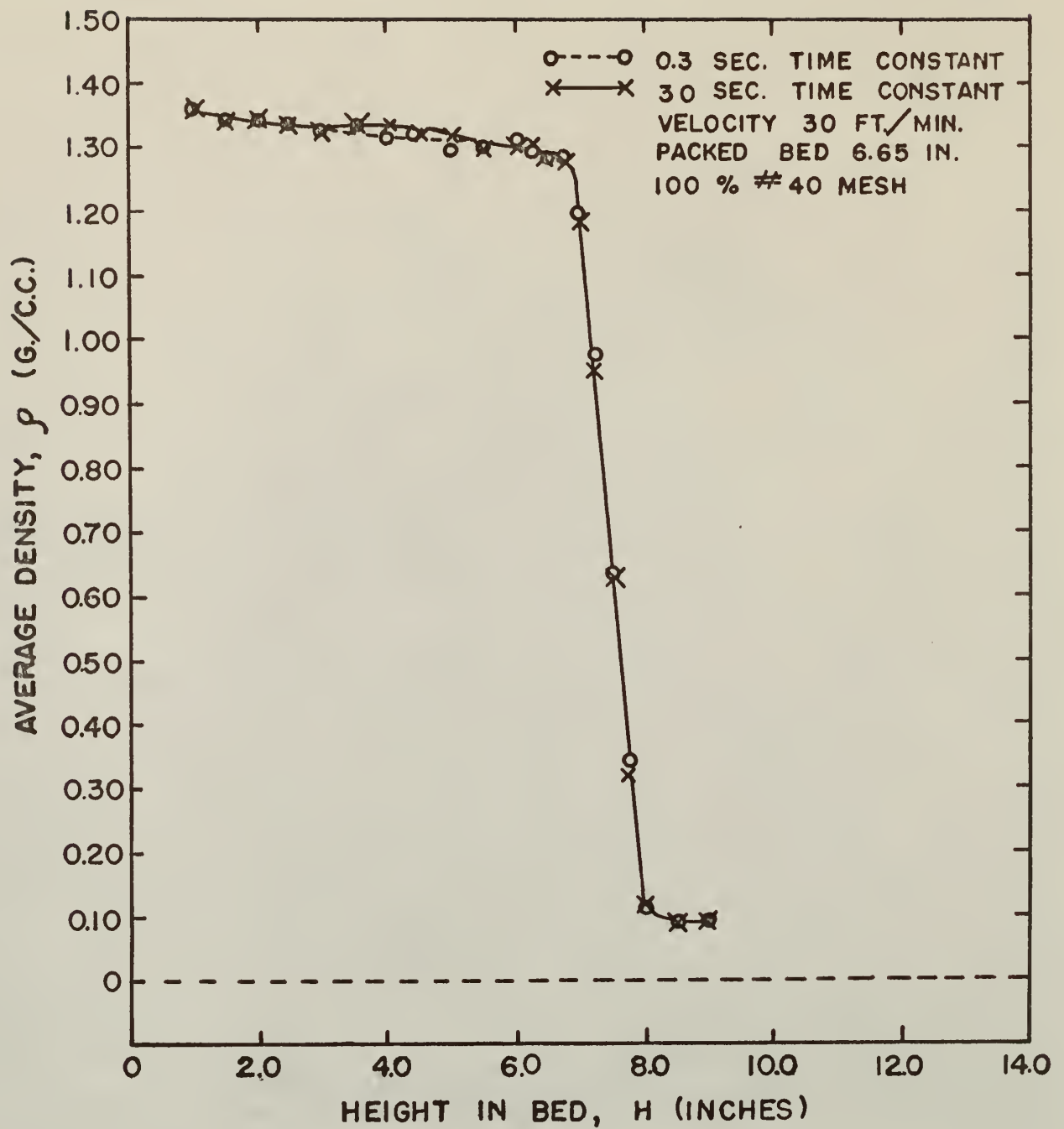


Fig. 34. Comparison of long and short time constants.

constant (0.3 second) in one particular run. The method used to calculate the line average density from the fluctuating CRM reading with a short-time constant was essentially the same as that used by Stewart (25). The axial density profiles obtained with both long-time constants and short-time constants are plotted in Fig. 34 for comparison. It can be seen that the results were almost identical, indicating that the sensitivity of the rate-count-meter was satisfactory. Considering the effort required to determine the axial density profile from the short-time constant data, we should be satisfied with the results obtained with a long-time constant for the purpose of the present studies. The density profiles were most directly obtained from the recording chart in long-time constant runs.

Reproducibility

The accuracy of the experimental result was determined by the consistency test. The consistency test was satisfactory if the instrument operated was stable and the power to detect the γ -ray was consistent throughout. Occasionally, the instrument became damped during the process of experiment, and the density profile obtained would be quite skewed even in the constant density zone. This is shown in Fig. 35. This deficiency could be improved by checking the calibration before and after taking the data, and by avoiding any mechanical shock to the γ -ray source. In fact, the reproducibility obtained was good, considering that the fluidizing state of beds changed constantly owing to the nature of bed itself. Installing more than one γ -ray source and scintillation probe to measure the solid density from several directions of the fluidizing column would have been necessary to improve the reproducibility to any larger extent.

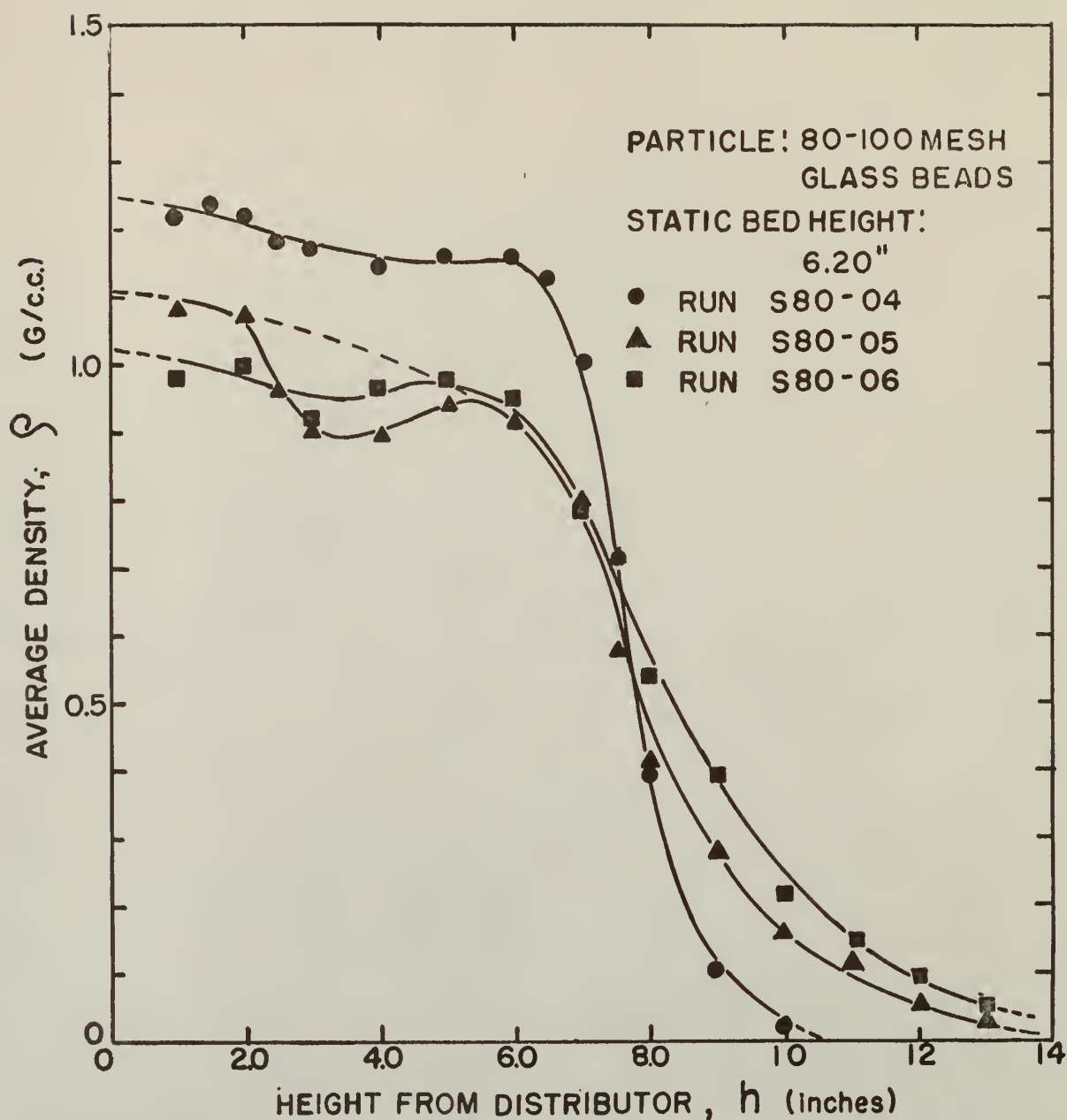


Fig. 35. The skewed density profile taken when the instrument was damped.

The Importance of the "Bed Density Distribution"
to the Understanding of Heat and Mass Transfer Mechanism
in Catalyst Fluidized Bed Reactor

In spite of the wide application and rapid development of fluidization techniques, a complete understanding of the fundamental factors governing the performance of fluidizing units has been lacking (especially for gas-solid systems.). One way for expressing the bed performance is in terms of various degrees of so-called "good" or "bad" fluidization based on some definite index. This index is a direct measure of the effectiveness of heat and mass transfer as well as the conversion of reactants in chemical reactors (15, 16, 18).

Heat and mass transfer correlations for packed beds are rather complete in the literatures (11, 28). There has been a trend during the past 15 years for investigators in the field of fluidization to use the analogy to the packed bed reactor in the study of fluidized bed reactors. But the difficulties encountered were due to the non-uniform particle distribution throughout the fluidized bed. The reacting gas flowing through the fluidized bed is not only a tortuous flow through the constant void space as in packed beds, but in addition it is affected by the impacts between particles and the interaction between particles and the flowing medium. A quantitative measurement of the complexity of the flowing pattern in the fluidized bed is therefore important. The mass and heat transfer in the fluidized bed is governed by the surface and the movement of particles in the bed. It is certain that the transfer is better in the locations where the solid particles are dense and are constantly circulating.

Mickley et.al. (16) in their study of heat transfer for gas fluidized beds concludes:

The local time-mean wall-to-bed heat transfer coefficient varies with the axial position of heat transfer surface, decreasing with increasing distance above the bottom of the bed. On the other hand, the local coefficient is not significantly influenced by total height of the bed.

Shirai et.al.(23) also drew the same conclusion in studying the axial distribution of heat transfer coefficients. The axial distribution of particles-to-fluid heat transfer coefficients obtained by them is almost identical in shape to the axial density profile obtained in this work.

The close relationship between the axial distribution of the heat transfer coefficient and the axial density profile suggests that the study of the density profile of fluidized beds will help in understanding heat and mass transfer mechanism in the fluidized bed, since the particle behavior is the major factor affecting the performance of fluidized beds.

CONCLUSION

The following significant conclusions have been drawn from the results of the present investigation:

1. With a long-time constant on the CRM, the γ -ray technique can be applied satisfactorily to the study of the axial density profile of the aggregative fluidized bed. The reproducibility is within the allowable error, except those at very high air velocities.
2. The data taken with a short-time constant is almost identical with the data using long-time constant.
3. Two distinct zones can be seen in the axial density profile, the constant density zone and the falling density zone. The line average density in the constant density zone decreases with increasing air velocity.

4. The ratio of the static bed height to the column diameter instead of the static bed height itself is the important variable influencing the fluidizing state.

5. Indices A and B can be used to describe aggregative fluidization. The greater the Index B, the more stable is the fluidizing bed.

6. Indices A and B are correlated by the following empirical equations:

$$|A| = 1.349 \times 10^{-5} (N_{Re})^{0.978} \left(\frac{D_p}{D_c}\right)^{-1.265} \left(\frac{L_{mf}}{D_c}\right)^{0.3469} \quad (22)$$

$$|B| = 5.492 \times 10^9 (N_{Re})^{-2.094} \left(\frac{D_p}{D_c}\right)^{3.045} \left(\frac{L_{mf}}{D_c}\right)^{0.3474} \quad (23)$$

7. Among the three significant operational variables, air velocity, static bed height, and particle size, air velocity has the most pronounced effect on the fluidizing state.

8. The bed height defined as the end point of the axial density profile corresponds to the visual maximum bed height. The more significant mean height of aggregative fluidized bed is defined as the cross point of the normalized density profile.

9. Pressure gradient ($\Delta P/L$) correlation yields the following equations:

$$f_K = \frac{D_p}{\rho_F V^2} \cdot \frac{g_c \Delta P}{L} \cdot \frac{\epsilon^3}{1 - \epsilon} =$$

$$(9.028 \times 10^5) \left(\frac{D_p V \rho_F}{\mu} \cdot \frac{1}{1 - \epsilon} \right)^{-0.945} \left(\frac{D_p}{D_c} \right)^{1.389} \left(\frac{L_{mf}}{D_c} \right)^{-0.224} \quad (25)$$

$$f_K = \frac{D_p}{\rho_F V^2} \cdot \frac{g_c \Delta P}{L} \cdot \frac{\epsilon^3}{1 - \epsilon} =$$

$$(8.80 \times 10^8) \left(\frac{D_p V \rho_F}{\mu} \cdot \frac{1}{1 - \epsilon} \right)^{-1.591} \left(\frac{D_p}{D_c} \right)^{2.126} \left(\frac{L_{mf}}{D_c} \right)^{-0.298} \quad (26)$$

In equation (25), L was determined by method (a), and in equation (26), by method (b) or (c). The second equation correlates the data better than the first one.

10. The heterogeneous distribution of fluidized bulk density in the aggregative fluidized beds has a direct influence on the heat transfer, mass transfer and local conversion of fluidized bed reactors.

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NOMENCLATURE

A	= cross-sectional area of the fluidized bed, l^2 .
A'	= part of cross-sectional area of the fluidized bed, l^2 .
A_r	= projected area of γ -beam path, l^2 .
A_0	= constant, dimensionless.
B_0	= constant, dimensionless.
A	= index of fluidized bed, dimensionless.
B	= index of fluidized bed, dimensionless.
d_{Al}	= thickness of spacer aluminum, l .
D_c	= inner diameter of fluidized column, l .
d_c	= wall thickness of fluidized column, l .
D_p	= diameter of particle, l .
D_p'	= width of packed particle bed in the direction of radiation beam, l .
D_s	= width of spacer, l .
f_K	= friction factor, dimensionless.
G	= volumetric air velocity, l^3/θ .
G_b	= volumetric bubble velocity, l^3/θ .
g_c	= conversion from force to mass, $ml/F\theta^2$.
G_{mf}	= volumetric velocity of minimum fluidization, l^3/θ .
h	= height above the distribution in bed, l .
I/I_0	= fraction of photons remaining in the beam after passage through absorber of thickness d , dimensionless.
L	= height of fluidized bed, l .
L_{mf}	= static (on-set) height of bed, l .
M	= total particle weight in bed, m .
M'	= bed weight obtained from integrated area (consistency test), m .

M_{s1}	= weight of particles fluidizing above L_{mf} , m.
M_{s2}	= weight of particles fluidizing below L_{mf} , m.
N_a	= Avagadro's number (6.025×10^{23} nuclei/g-mol).
N_{Re}	= Reynolds number, $V \rho_{FDp} / \mu$, dimensionless.
N_{Re}'	= modified Reynolds number, $V \rho_{FDp} / \mu \cdot 1/1 - \epsilon$, dimensionless.
ΔP	= pressure drop through fluidized bed, F/l ²
t_b	= the residence time of bubbles in the bed below L_{mf} , θ .
V	= superficial air velocity, l/ θ .
\bar{V}_b	= average linear velocity of bubbles, l/ θ .
V_{mf}	= minimum velocity of fluidization, l/ θ .
Vol_b	= the volume occupied by bubbles below L_{mf} , l ³ .
Vol_{s1}	= volume of particles fluidizing above L_{mf} , l ³ .
Vol_{s2}	= volume of particles fluidizing below L_{mf} , l ³ .

Greek Letters

α'	= constant.
β'	= constant.
ϵ_{mf}	= porosity of the fluidized bed at on-set fluidization.
ρ	= fluidized bulk density.
ρ_{Al}	= density of aluminum.
ρ_c	= density of the fluidizing column material.
ρ_d	= fluidized bulk density at the position corresponding to the static bed height.
$\bar{\rho}_d$	= average line density in the constant density zone.
ρ_F	= density of fluid.
ρ_{mf}	= fluidized bulk density at on-set fluidization.
ρ_{pa}	= bulk density of bed at packed state.

ρ_s	= density of solid particle.
ϕ	= constant.
σ	= microscopic absorption cross-section.
μ	= viscosity of fluid.
μ_m	= absorption coefficient.

APPENDIX

APPENDIX I

Table 3. Summary of the experimental and calculated data.

Run No.	P40-01	P40-02	P40-03	P40-04	P40-05	P40-06
Part. Size, D_p (inch)	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153
Static Bd. Ht. (inch)	2.75	2.75	2.75	2.75	6.20	6.20
Air Flow Rate (ft./min.)	30	60	90	110	30	60
Exp'd. Bd. Ht., L , (inch)						
(by method a)	3.516	4.872	5.904	7.608	8.436	10.608
(by method b)	2.952	3.408	3.552	3.900	6.948	7.104
Pres. Drop per Unit Bd. Ht.						
(by method a)	56.075	40.468	33.394	25.915	59.255	47.123
$\Delta P/L$						
(by method b)	66.788	57.852	55.506	50.554	71.945	70.366
Modified Reynold's No.						
$N'_{Re} = \frac{D_p \rho_F V}{\mu(1-\epsilon)}$						
(a)	8.238	22.844	41.334	65.186	8.916	22.369
(b)	6.914	15.967	24.889	33.362	7.352	14.955
Reynold's Number						
$N_{Re} = \frac{D_p \rho_F V}{\mu}$						
	3.872	7.744	11.574	14.145	3.941	7.852
$\left[\frac{D_p}{\eta_c} \text{ ratio} \right] \times 10^3$						
	3.848	3.848	3.848	3.848	3.848	3.848
Friction factor						
$f_K = \frac{g_c \Delta P}{L} \cdot \frac{D_p}{\rho_F V^2} \cdot \frac{\epsilon^3}{1-\epsilon}$						
(a)	39.013	18.931	10.904	9.400	50.267	19.876
(b)	22.314	8.946	4.477	3.736	28.938	7.779
Index A	-0.054	-0.105	-0.172	-0.207	-0.069	-0.145
Index B	-16.780	-3.437	-1.391	-0.813	-21.459	-3.150

Note: Index A is the correlated intercept for density falling-zone profile.

Index B is the correlated slope for density falling-zone profile.

Table 3. Summary of the experimental and calculated data. (Cont.)

Run No.	P40-07	P40-08	P40-09	P40-10	P40-11	P40-12
Part. Size, D_p (inch)	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153
Static Bd. Ht. (inch)	6.20	6.20	9.10	9.10	9.10	9.10
Air Flow Rate (ft ³ /min.)	90	110	30	60	90	110
Exp'd Bd. Ht., L , (inch)						
(by method a)	12.120	14.568	11.148	14.556	17.748	21.936
(by method b)	7.140	7.200	9.552	10.380	10.464	11.124
Pres. Drop per Unit Bd. Ht.						
(by method a)	41.244	34.313	67.004	51.317	42.087	34.052
$\Delta P/L$						
(by method b)	70.011	69.427	78.199	71.962	71.384	67.149
Modified Reynold's No.						
$N'_{Re} = \frac{D_p \int F V}{\mu (1 - \epsilon)}$						
(a)	38.023	55.912	7.958	20.786	38.063	56.907
(b)	22.435	27.632	6.845	14.830	22.459	28.859
Reynold's Number						
$N_{Re} = \frac{D_p \int F V}{\mu}$						
$[D_p/D_c \text{ ratio}] \times 10^3$						
(a)	11.711	14.314	3.915	7.816	11.723	14.170
(b)	3.848	3.848	3.848	3.848	3.848	3.848
Friction factor						
$f_K = \frac{g \Delta P}{L} \cdot \frac{D_p}{\int F V^2} \cdot \frac{\epsilon^3}{1 - \epsilon}$						
(a)	10.741	8.945	38.782	18.042	10.949	9.482
(b)	3.546	2.432	23.283	7.862	3.612	2.952
Index A	-0.242	-0.279	-0.0678	-0.153	-0.262	-0.291
Index B	-1.645	-1.133	-36.960	-5.203	-1.213	-0.815

Note: Index A is the correlated intercept for density falling-zone profile.
 Index B is the correlated slope for density falling-zone profile.

Table 3. Summary of the experimental and calculated data. (Cont.)

Run No.	S40-01	S40-02	S40-03	S40-05	S40-06	S40-07
Part. Size, D _p (inch)	0.0153	0.0153	0.0153	0.0153	0.0153	0.0153
Static Bd. Ht. ^p (inch)	3.25	3.25	3.25	6.65	6.65	6.65
Air Flow Rate (ft./min.)	30	60	90	30	60	90
Exp'd. Bd. Ht., L, (inch)						
(by method a)	4.875	5.982	7.580	8.976	12.372	14.964
(by method b)	3.652	3.780	4.000	7.176	7.764	7.932
Pres. Drop per Unit Bd. Ht.						
(by method a)	53.327	43.458	34.296	59.360	43.066	35.606
Δ P/L						
(by method b)	71.185	68.774	64.992	74.249	68.626	67.172
Modified Reynold's No.						
$N'_{Re} = \frac{D_p \rho_F V}{\mu (1 - \epsilon)}$						
(a)	9.478	23.766	44.404	8.601	20.059	43.321
(b)	7.246	15.020	23.438	6.871	14.918	22.735
Reynold's Number						
$N_{Re} = \frac{D_p \rho_F V}{\mu}$	3.871	7.750	11.898	3.827	7.683	11.567
$[D_p/D_c \text{ ratio}] \times 10^3$	3.848	3.848	3.848	3.848	3.848	3.848
Friction factor						
$f_K = \frac{g_c \Delta P}{L} \cdot \frac{D_p}{\rho_F V^2} \cdot \frac{\epsilon^3}{1 - \epsilon}$						
(a)	59.471	22.513	13.564	50.673	14.618	12.872
(b)	29.761	8.391	4.541	25.752	8.414	3.829
Index A	-0.046	-0.083	-0.158	-0.069	-0.122	-0.215
Index B	-10.391	-3.327	-1.033	-23.996	-3.150	-1.150

Note: Index A is the correlated intercept for density falling-zone profile.
Index B is the correlated slope for density falling-zone profile.

Table 3. Summary of the experimental and calculated data. (Cont.)

Run No.	S40-09	S40-10	P80-04	P80-05	P80-06
Part. Size, D_p (inch)	0.0153	0.0153	0.00642	0.00642	0.00642
Static Bd. Ht. H_p (inch)	9.40	9.40	6.10	6.10	6.10
Air Flow Rate (ft/min.)	30	60	30	60	90
Exp'd. Bd. Ht., L , (inch)					
(by method a)	12.290	15.490	11.004	13.632	16.356
(by method b)	9.750	10.232	7.380	7.500	7.752
Pres. Drop per Unit Bd. Ht.					
(by method a)	61.235	48.578	39.9346	32.2360	26.8672
$\Delta P/L$					
(by method b)	77.178	73.543	59.5389	58.5734	56.7259
Modified Reynold's No.					
$N_{Re} = \frac{D_p \rho_F V}{\mu (1 - \epsilon)}$					
(a)	8.487	21.410	5.058	12.532	22.254
(b)	6.741	14.145	3.394	6.899	10.542
Reynold's Number					
$N_{Re} = \frac{D_p \rho_F V}{\mu}$	3.900	7.838	1.639	3.278	4.854
$\left[\frac{D_p}{D_c} \text{ ratio} \right] \times 10^3$	3.848	3.848	1.6140	1.6140	1.6140
Friction factor					
$f_K = \frac{g_{c\Delta P}}{L} \cdot \frac{D_p}{\rho_F V^2} \cdot \frac{\epsilon^3}{1 - \epsilon}$					
(a)	46.036	18.753	34.756	11.330	6.056
(b)	21.806	6.577	15.573	4.069	1.991
Index A	-0.080	-0.175	-	-	-
Index B	-25.685	-2.925	-	-	-

Note: Index A is the correlated intercept for density falling-zone profile.
Index B is the correlated slope for density falling-zone profile.

Table 3. Summary of the experimental and calculated data. (Cont.)

Run No.	P80-07	P80-08	S80-01	S80-02	S80-03	S80-04
Part. Size, D_p (inch)	0.00642	0.00642	0.00642	0.00642	0.00642	0.00642
Static Bd. Ht. (inch)	8.75	8.75	3.25	3.25	3.25	6.20
Air Flow Rate (ft./min.)	30	60	30	60	90	30
Exp'd. Bd. Ht., L , (inch)						
(by method a)	13.356	15.300	6.348	7.308	7.860	10.236
(by method b)	10.020	10.284	4.164	4.224	4.356	7.224
Pres. Drop per Unit Bd. Ht.						
(by method a)	51.258	44.745	37.075	32.220	29.943	40.317
$\Delta P/L$						
(by method b)	68.339	66.592	56.577	55.734	54.044	62.110
Modified Reynold's No.						
$N^* Re = \frac{D_p \int_F V}{\mu (1 - \epsilon)}$	(a) (b)					
	4.251 3.191	9.741 6.550	5.374 3.523	12.373 7.157	19.964 11.065	4.480 3.159
Reynold's Number						
$N_{Re} = \frac{D_p \int_F V}{\mu} \times 10^3$						
	1.628 1.614	3.757 1.614	1.608 1.614	3.217 1.614	4.825 1.614	1.586 1.614
Friction factor						
$f_K = \frac{g \Delta P}{L} \cdot \frac{D_p}{\int_F V^2} \cdot \frac{\epsilon^3}{1 - \epsilon}$	(a) (b)					
	28.881 14.448	9.069 3.910	39.673 18.510	11.682 4.814	5.588 2.299	28.991 14.429
Index A	-	-	-0.079	-0.167	-0.208	-0.106
Index B	-	-	-2.142	-1.406	-1.135	-4.765

Note: Index A is the correlated intercept for density falling-zone profile.
Index B is the correlated slope for density falling-zone profile.

Table 3. Summary of the experimental and calculated data. (Cont.)

Run No.	S80-05	S80-06	S80-07	S80-08
Part. Size, D_p (inch)	0.00642	0.00642	0.00642	0.00642
Static Bd. Ht. (inch)	6.20	6.20	9.35	9.35
Air Flow Rate (ft./min.)	60	90	30	60
Exp'd. Bd. Ht., L , (inch)	12.276	14.472	13.764	17.184
(by method a)	7.320	7.632	10.800	11.316
(by method b)				
Pres. Drop per Unit Bd. Ht.	36.550	31.004	45.6415	36.5579
(by method a)				
$\Delta P/L$	61.296	58.790	58.1676	55.5270
(by method b)				
Modified Reynold's No.				
$N'_{Re} = \frac{D_p \int_F V}{\mu (1 - \epsilon)}$	(a) 10.830	19.064	4.028	10.0506
	(b) 6.452	10.034	3.163	6.6268
Reynold's Number				
$N_{Re} = \frac{D_p \int_F V}{\mu}$	3.195	4.766	1.600	3.200
$[D_p/D_c \text{ ratio}] \times 10^3$	1.614	1.614	1.614	1.614
Friction factor				
$f_K = \frac{\Delta P}{L} \cdot \frac{D_p}{\int_F V^2} \cdot \frac{\epsilon^3}{1 - \epsilon}$	(a) 10.174	5.479	23.555	8.506
	(b) 3.737	1.865	12.980	3.720
Index A	-0.166	-0.199	-0.098	-0.163
Index B	-2.185	-1.238	-7.924	-2.161

Note: Index A is the correlated intercept for density falling-zone profile.

Index B is the correlated slope for density falling-zone profile.

Table 3. Summary of the experimental and calculated data. (Cont.)

Run No.	EX-01	EX-02	EX-03	EX-04	EX-05	EX-06
Part. Size, D_p (inch)	0.00906	0.00906	0.00906	0.00906	0.00906	0.00906
Static Bd. Ht. (inch)	5.0	5.0	5.0	6.0	6.0	6.0
Air Flow Rate (ft./min.)	35.6	55.4	69.3	35.6	55.4	69.3
Exp'd Bd. Ht., L , (inch)						
(by method a)	7.50	9.20	11.80	9.80	11.40	12.30
(by method b)	5.87	6.39	6.6	7.22	7.71	8.00
Pres. Drop per Unit Bd. Ht.						
(by method a)	60.890	49.636	38.703	55.917	47.955	44.554
$\Delta P/L$						
(by method b)	77.798	71.467	69.195	75.901	71.078	68.498
Modified Reynold's No.						
(a)	6.927	13.223	21.271	7.563	13.654	18.478
(b)	5.421	9.184	11.989	5.572	9.234	12.018
Reynold's Number						
(a)	2.770	4.309	-	-	-	-
(b)	2.278	2.278	2.278	2.278	2.278	2.278
Friction factor						
(a)	32.923	19.008	16.247	38.248	19.866	13.936
(b)	18.310	9.548	6.590	19.619	9.706	6.748
Index A	-0.1319	-0.1420	-0.1717	-0.1029	-0.1630	-0.1988
Index B	-5.600	-2.4500	-2.035	-6.1900	-2.850	-2.1420

Note: Index A is the correlated intercept for density falling-zone profile.
Index B is the correlated slope for density falling-zone profile.

APPENDIX II

(A) Derivation of the Dimensionless Equation for Correlating Pressure Gradient in the Aggregative Fluidized Bed----Friction Factor, f_K

The pressure gradient, $\Delta P/L$, has been found experimentally to be influenced by the following properties of column, fluid, and particles: column diameter, D_C , fluid density, ρ_F , viscosity, μ , velocity, V , particle diameter, D_p , and static bed height, L_{mf} . The primary relationship existing between the pressure gradient and the variables is (6, 11, 28):

$$\frac{\Delta P}{L} = \phi (D_C)^h (\rho_F)^i (\mu)^j (V)^k (D_p)^m (L_{mf})^n (g_c)^p \quad (27)$$

where ϕ , h , i , j , k , m , n , q , are unknown constants, and g_c is the conversion constant between force and mass.

Equation (27) is rewritten using the dimensions of each variable as:

$$\frac{F}{L^3} = \phi (1)^h \left(\frac{m}{1}\right)^i \left(\frac{m}{1-\theta}\right)^j \left(\frac{1}{\theta}\right)^k (1)^m (1)^n \left(\frac{ml}{F\theta^2}\right)^q \quad (28)$$

Summing the exponents of equation (28),

$$\sum F, \quad 1 = -q$$

$$\sum 1, \quad -3 = h - 3i - j + k + m + n + q$$

$$\sum m, \quad 0 = i + j + q$$

$$\sum \theta, \quad 0 = -j - k - 2q$$

To solve the above four equations for seven unknowns, the Bridgman Pi-Theorem* is used. Choosing h , j , n , as three unknowns, then,

*Bridgman's Pi-theorem: If there are N number of equations to solve for K unknowns, the number of dimensionless groups to be obtained is $N - K + 1$, and $N - K$ of unknowns retain numerically unsolved.

$$q = -1$$

$$i = -j - q = 1 - j$$

$$k = -j - 2q = 2 - j$$

$$m = -3 - h + 3i + j - k - n - q$$

$$= -h - j - n - 1$$

Substituting the above values into equation (27):

$$\left(\frac{\Delta P}{L}\right) = \phi (D_c)^h (\rho_F)^{1-j} (\mu)^j (V)^{2-j} (D_p)^{-1-h-j-n} (L_{mf})^n (g_c)^{-1} \quad (29)$$

and rearranging:

$$\frac{g_c \Delta P}{L} = \phi \left(\frac{\mu}{\rho_F V D_p}\right)^j \left(\frac{D_c}{D_p}\right)^h \left(\frac{L_{mf}}{D_p}\right)^n \left(\frac{\rho_F V^2}{D_p}\right)$$

or,

$$\left(\frac{g_c \Delta P}{L} \cdot \frac{D_p}{\rho_F V^2}\right) = \phi \left(\frac{\rho_F V D_p}{\mu}\right)^{-j} \left(\frac{D_p}{D_c}\right)^{-h-n} \left(\frac{L_{mf}}{D_c}\right)^n$$

Let $-j = a$, $-h - n = b$, $n = c$, then:

$$\left(\frac{g_c \Delta P}{L} \cdot \frac{D_p}{\rho_F V^2}\right) = \phi \left(\frac{\rho_F V D_p}{\mu}\right)^a \left(\frac{D_p}{D_c}\right)^b \left(\frac{L_{mf}}{D_c}\right)^c \quad (30)$$

Based on the theoretical consideration (for the hydraulic radius of column and the wetted surface of particles, etc.), the friction factor, f_K , derived for the packed bed has been modified by multiplying the dimensionless factor, $\epsilon^{3/1 - \epsilon}$ (6), and the Reynolds number, $\rho_F V D_p / \mu$, has been modified by multiplying by $1/1 - \epsilon$ (where ϵ is the void fraction of the fluid-particles bed.) The dimensionless equation then becomes:

$$\begin{aligned} f_K &= \left(\frac{g_c \Delta P}{L} \cdot \frac{D_p}{\rho_F V^2} \cdot \frac{\epsilon^3}{1 - \epsilon}\right) \\ &= \phi \left(\frac{\rho_F V D_p}{\mu} \cdot \frac{1}{1 - \epsilon}\right)^a \left(\frac{D_p}{D_c}\right)^b \left(\frac{L_{mf}}{D_c}\right)^c \end{aligned} \quad (24)$$

(B) Derivation of the Dimensionless equation for Correlating Indices A and B.

The results of the previous and the present investigation indicate that (5, 25):

$$|A| = \alpha' (D_c)^h (\rho_F)^i (\mu)^j (V)^k (D_p)^m (L_{mf})^n \quad (31)$$

$$/ = \alpha' (1)^h (m/1^3)^i (m/1 - \Theta)^j (1/\Theta)^k (1)^m (1)^n$$

$$\sum m, 0 = i + j$$

$$\sum 1, 0 = h - 3i - j + k + m + n$$

$$\sum \Theta, 0 = -j - k$$

Holding h, j, n , as unknown constants

$$i = -j$$

$$k = -j$$

$$m = -h + 3i + j - k - n = -h - j - n$$

substituting these into equation (31),

$$|A| = \alpha' (D_c)^h (\rho_F)^{-j} (\mu)^j (V)^{-j} (D_p)^{-h-j-n} (L_{mf})^n \quad (32)$$

Rearranging:

$$\begin{aligned} |A| &= \alpha' (\mu / \rho_F V D_p)^j (D_c / D_p)^h (L_{mf} / D_p)^n \\ &= \alpha' (\rho_F V D_p / \mu)^{-j} (D_p / D_c)^{-h-j-n} (L_{mf} / D_c)^n \end{aligned}$$

Let $-j = a$, $-h - n = b$, $n = c$, then:

$$|A| = \alpha' (\rho_F V D_p / \mu)^a (D_p / D_c)^b (L_{mf} / D_c)^c \quad (20)$$

By the similar method, the dimensionless equation for correlating index B can be obtained as:

$$|B| = \beta' (\rho_F V D_p / \mu)^{a'} (D_p / D_c)^{b'} (L_{mf} / D_c)^{c'} \quad (21)$$

AXIAL SOLID-DENSITY DISTRIBUTION IN
GAS SOLID FLUIDIZED BEDS

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γ -ray attenuation method was applied to determine the axial fluidizing bulk density profile for an air-solid aggregative fluidized bed. Two distinct density zones, constant and falling, were observed except for the bed fluidizing at very high air velocity.

The average density of the constant zone $\bar{\rho}_d$, was correlated as a function of the operating variables as:

$$\bar{\rho}_d = \alpha - \beta (V - V_{mf})$$

where α and β are the constants depending on the particle diameter only, V and V_{mf} are fluidizing gas velocity and minimum fluidizing velocity of the particles.

The density, ρ , in the falling zone was found to be approximated by the following equation, which is characterized by the two indices A and B:

$$\text{Log} (\rho / \rho_{pa}) = A + B (h/L_{mf} - 1)^2$$

The effects of operational variables, such as air velocity, V , static bed height, L_{mf} , particle size, D_p , on these indices were investigated. They were expressed as functions of dimensionless groups as:

$$|A| = 1.349 \times 10^{-5} \left(\frac{D_p V \rho_F}{\mu} \right)^{0.978} (D_p/D_c)^{-1.265} (L_{mf}/D_c)^{0.3469}$$

$$|B| = 5.492 \times 10^9 \left(\frac{D_p V \rho_F}{\mu} \right)^{-2.094} (D_p/D_c)^{3.045} (L_{mf}/D_c)^{0.3474}$$

The effects of particle composition in beds containing a mixture of two sizes of particles (40-45 mesh and 80-100 mesh glass beads) on fluidization characteristics were also studied. The results showed that the values of

indices A and B were significantly changed only when the mixture contained less than 10 weight percent of one component.

Three different ways to define the height of a fluidized bed were considered:

- (a) End point of the density profile.
- (b) Cross point of the normalized density profile with that of an ideal fluidized bed.
- (c) Maximum point of axial instability index distribution curve.

The height defined in (a) corresponded to the maximum bed height as visually observed, while the other two are the average of the fluctuating bed height.

Pressure gradient was correlated in the form of a friction factor. The heights defined in (b) and (c) gave better correlation than that defined in (a). The friction factor obtained is given as follows:

$$f_K = \left(\frac{D_P}{\rho_{FV}^2} \cdot \frac{g_c \Delta P}{L} \cdot \frac{\epsilon^3}{1 - \epsilon} \right) =$$

$$(8.80 \times 10^7) \left(\frac{D_P V \rho_F}{\mu} \cdot \frac{1}{1 - \epsilon} \right)^{-1.591} \left(\frac{D_P}{D_C} \right)^{2.126} \left(\frac{L_{mf}}{D_C} \right)^{-0.298}$$

The heterogeneous distribution of fluidized bulk density in the aggregative fluidized beds has direct influences on the transport and rate processes. Therefore, the knowledge of the density distribution in the aggregative fluidized beds cannot be over-emphasized. The works of previous investigators on the transport and rate processes in fluidized beds were re-examined in the light of the results obtained in this investigation.

